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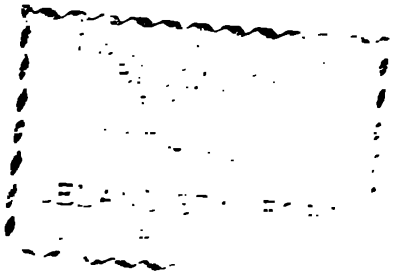

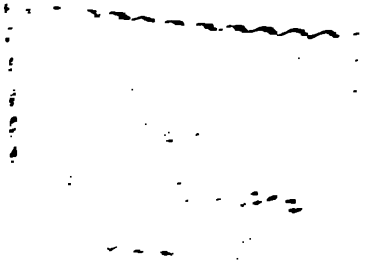

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UNITING ORAL AND WRITTEN EXERCISES

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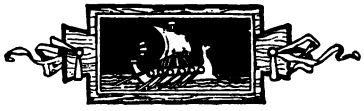

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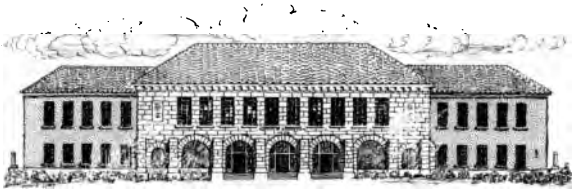

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PREFACE.

For several years past there has been an increasing demand for a **two-book** series of Arithmetics, and various attempts to meet this demand have been made, but so far unsuccessfully.

The most common device has been the selection of two books from a series composed of three or more books, and the presentation of the treatises, thus selected, as a complete two-book series. In several instances a thin little primary, with simple lessons for the first two or three years of school instruction, has been united with an advanced treatise, nearly every page of which presents matter too difficult for children under twelve years of age. In other instances the primary lessons have been wholly discarded, and the second and a higher book have been made a two-book series.

Both of these expedients have proved unsatisfactory, for the obvious reason that the first book either *ends too low* or *begins too high*. The use of a thin primary as a first book brings the pupils to the second or higher book before they have the capacity or the training requisite to master such a treatise, and the use of a second book for a first begins the course at too high a point, and makes necessary several years of oral instruction, much of which is poor, or is given with unnecessary labor on the part of both teachers and pupils. The lessons in number, adapted to the first two years of school life, can best be given orally, but exclusive oral instruction in the third, fourth, and fifth years involves an amount of blackboard and slate work which is a severe tax on the eyes and nerves of little children.

The Elementary Arithmetic aims to avoid both of these errors. It covers the ground of the first two books of the author's three-book series, but does not present the first lessons so fully. It takes it for granted that pupils who have been in school two or three years do not need to spend months in learning to count, read, and write numbers from 1 to 100.

The first sixty-two pages present a thorough drill with small numbers, in which the converse processes of addition and sub-

traction, and of multiplication and division are combined. These first lessons embody the more important principles of what is known as the "Grube Method," but they are not carried to the extreme of useless repetition and mechanism.

The next sixty-two pages are devoted to the fundamental operations with larger integral numbers, and the written processes are more fully developed. These are followed by an elementary course in Fractions, common and decimal, United States Money, Denominate Numbers, Mensuration, and Percentage, including Simple Interest and Bank Discount.

The work presents thorough drills in all elementary processes, with both integral and fractional numbers, and includes those practical applications of numbers which are most frequently used in business and common life. All these operations are presented in a simple manner, and the problems are within the comprehension of young pupils. The work thus prepares pupils for the successful study of the second book (the New Complete Arithmetic) and, at the same time, it presents a practical **Short Course in Arithmetic** for the large number of pupils who do not attend school long enough to complete a higher and larger treatise. Many pupils leave school before they reach, in these higher books, such practical subjects as percentage and interest.

The characteristics that have given the author's Arithmetics wide and successful use in the schools of the country, are preserved in the new series. These include:

1. A special adaptation, in matter and method, to the grade of pupils for which each book is designed.

2. A practical union of oral and written exercises in a natural and philosophic system of instruction.

3. A true and practical embodyment of the inductive method.

4. The great variety and practical character of the problems, the number of business problems being greatly increased in the New Series.

The Elementary Arithmetic is not a manual for teachers, with "model lessons" for their special use, but it is a *drill book for pupils.* Instruction specially designed for teachers is embodied in the author's "Manual of Arithmetic," which is a teacher's hand-book, with suggestions and directions, illustrative solutions, and classified problems for dictation.

PURDUE UNIVERSITY,
Lafayette, Ind., January 3, 1883.

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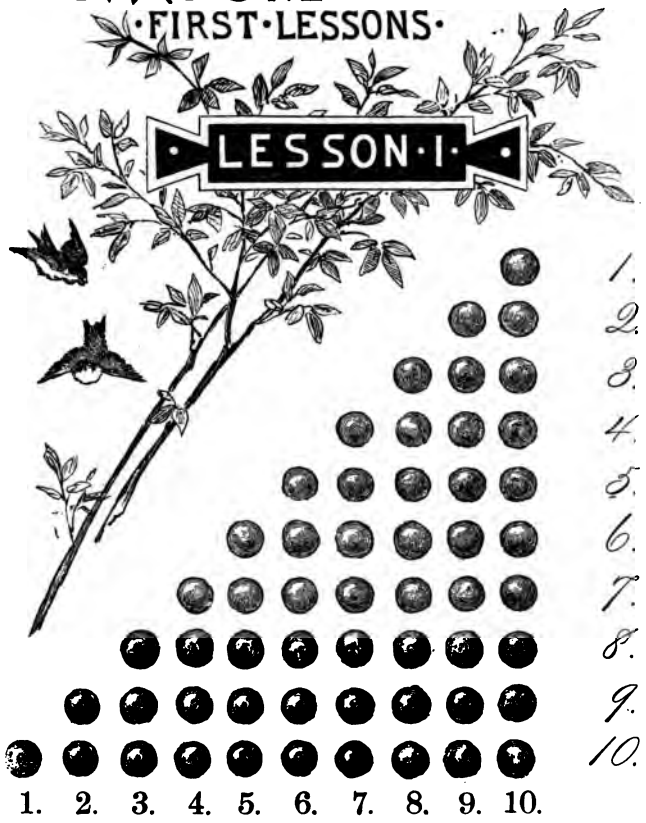
ELEMENTARY ARITHMETIC.

PART ONE.

FIRST LESSONS.

LESSON I.



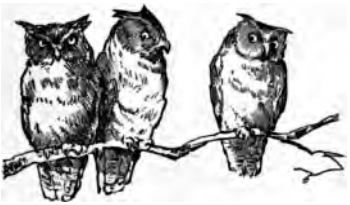
COUNT the balls in each row from left to right and from right to left. Count also the rows up and down.

How many balls are in the lowest row? How many in the third row from the top? How many in the fifth row? How many in the seventh?

To TEACHERS.—Make the preceding table on the blackboard, using circles, squares, triangles, or other simple forms, and drill the pupils in *rapid counting*, thus (pointing to the forms): One: one, two; two, one: one, two, three; three, two, one, etc.—each row being counted in two directions. Count first the horizontal rows, and then the vertical. The drill should be continued until rapidity and accuracy are secured.

LESSON II.

1. How many owls are 2 owls and 1 owl? How many owls are 1 owl and 2 owls?



How many are 2 and 1? 1 and 2?

One owl taken from 3 owls leaves how many owls? 2 owls from 3 owls?

One from 3 leaves how many? 2 from 3?

Make the figures one, two, and three on your slate, thus: 1 2 3.

2. How many birds are three birds and 1 bird? 1 bird and 3 birds? 2 birds and 2 birds?

How many are three and 1? 1 and 3? 2 and 2?

One bird from 4 birds leaves how many birds? 3 birds from 4 birds? 2 birds from 4 birds?

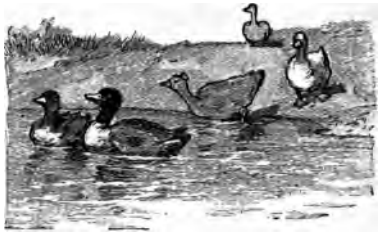
One from 4 leaves how many? 3 from 4? 2 from 4?

Make the figure four on your slate, thus: 4.

Make the figure 4 four times.

3. How many ducks are 4 ducks and 1 duck? 1 *duck and 4 ducks?* 3 ducks and 2 ducks? 2 ducks *and 3 ducks?*

How many are 4 and 1? 1 and 4? 3 and 2? 2 and 3?



One duck from 5 ducks leaves how many ducks? 4 ducks from 5 ducks? 2 ducks from 5 ducks? 3 ducks from 5 ducks?

One from 5 leaves how many? 4 from 5? 2 from 5? 3 from 5?

Make the figure five on your slate, thus: 5.

Make the figure 5 five times.

4. How many chicks are 5 chicks and 1 chick? 4 chicks and 2 chicks? 2 chicks and 4 chicks? 3 chicks and 3 chicks?

How many are 5 and 1? 1 and 5? 4 and 2? 2 and 4? 3 and 3?

One chick from 6 chicks leaves how many? 5 chicks from 6 chicks? 2 chicks from 6 chicks? 4 chicks from 6 chicks? 3 chicks from 6 chicks?

One from 6 leaves how many? 5 from 6? 2 from 6? 4 from 6? 3 from 6?

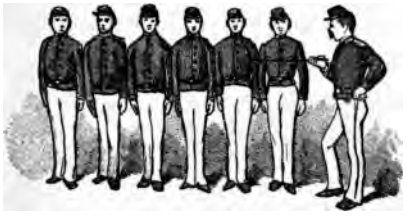
Make the figure six on your slate, thus: 6.

Make the figure 6 six times.

5. How many men are 6 men and 1 man? 5 men and 2 men? 2 men and 5 men? 4 men and 3 men? 3 men and 4 men?

How many are 6 and 1? 5 and 2? 2 and 5? 4 and 3? 3 and 4?

One man from 7 men leaves how many? 2 men from 7 men? 5 men from 7 men? 3 men from 7 men? 4 men from 7 men?



One from 7 leaves how many? 6 from 7? 2 from 7? 5 from 7? 3 from 7? 4 from 7? 6 from 7? 5 from 7? 3 from 7?

Make the figure seven on your slate, thus: 7.

Make the figure 7 seven times.

6. How many cups are 7 cups and 1 cup? 6 cups and 2 cups? 2 cups and 6 cups? 3 cups and 5 cups? 5 cups and 3 cups? 4 cups and 4 cups?

How many are 7 and 1? 6 and 2? 2 and 6? 3 and 5? 5 and 3? 4 and 4?

One cup from 8 cups leaves how many? 2 cups from 8 cups? 6 cups from 8 cups? 3 cups from 8 cups? 5 cups from 8 cups? 4 cups from 8 cups?

One from 8 leaves how many? 2 from 8? 6 from 8? 3 from 8? 5 from 8? 4 from 8?

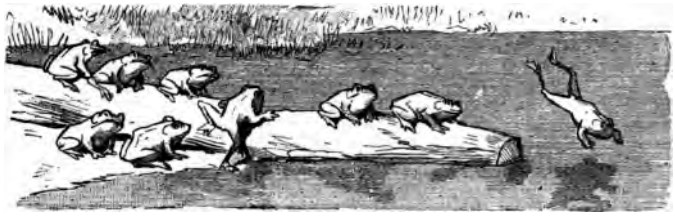
Make the figure eight on your slate, thus: 8.

Make the figure 8 eight times.

7. How many are 8 frogs and 1 frog? 1 frog and 8 frogs? 7 frogs and 2 frogs? 2 frogs and 7 frogs? 6 *frogs and 3 frogs*? 3 frogs and 6 frogs? 5 frogs and 4 *frogs? 4 frogs and 5* frogs?

How many are 8 and 1? 1 and 8? 7 and 2? 2 and 7? 6 and 3? 3 and 6? 5 and 4? 4 and 5?

One frog from 9 frogs leaves how many frogs? 2 frogs from 9 frogs? 7 frogs from 9 frogs? 3 frogs from 9 frogs? 6 frogs from 9 frogs? 4 frogs from 9 frogs? 5 frogs from 9 frogs?



One from 9 leaves how many? 2 from 9? 7 from 9? 3 from 9? 6 from 9? 4 from 9? 5 from 9?

Make the figure nine on your slate, thus: *9*.

Make the figure 9 nine times.

8. How many books are 9 books and 1 book? 8 books and 2 books? 7 books and 3 books? 6 books and 4 books? 5 books and 5 books?

How many are 9 and 1? 8 and 2? 7 and 3? 6 and 4? 5 and 5?

One book from 10 books leaves how many? 2 books from 10 books? 3 books from 10 books? 4 books from 10 books? 5 books from 10 books? 6 books from 10 books?

One from 10 leaves how many? 2 from 10? 3 from 10? 4 from 10? 5 from 10? 6 from 10?

Make the figures that stand for ten, thus: *10*.

Make the figures standing for 10 ten times.

The figure 0 is called *naught* or *cipher*. It denotes the *absence* of number.

How many are:

1 and 2?	2 and 1?	1 from 3?	2 from 3?
1 and 3?	3 and 1?	1 from 4?	3 from 4?
2 and 2?	———	2 from 4?	———
1 and 4?	4 and 1?	1 from 5?	4 from 5?
2 and 3?	3 and 2?	2 from 5?	3 from 5?
1 and 5?	5 and 1?	1 from 6?	5 from 6?
2 and 4?	4 and 2?	2 from 6?	4 from 6?
3 and 3?	———	3 from 6?	———
1 and 6?	6 and 1?	1 from 7?	6 from 7?
2 and 5?	5 and 2?	2 from 7?	5 from 7?
3 and 4?	4 and 3?	3 from 7?	4 from 7?
1 and 7?	7 and 1?	1 from 8?	7 from 8?
2 and 6?	6 and 2?	2 from 8?	6 from 8?
3 and 5?	5 and 3?	3 from 8?	5 from 8?
4 and 4?	———	4 from 8?	———
1 and 8?	8 and 1?	1 from 9?	8 from 9?
2 and 7?	7 and 2?	2 from 9?	7 from 9?
3 and 6?	6 and 3?	3 from 9?	6 from 9?
4 and 5?	5 and 4?	4 from 9?	5 from 9?

To TEACHERS.—Pupils may be easily taught to give the analysis and synthesis of each of the nine digits, as above, and also the number 10, without questions, thus:

3 is 1 and 2, or 2 and 1: 3 less 1 is 2; 3 less 2 is 1.

LESSON III.

1. How many books are 10 books and 1 book? 10 books and 2 books? 10 books and 3 books? 10 books and 4 books? 10 books and 5 books?

2. How many are 10 and 1? 10 and 2? 10 and 3? 10 and 4? 10 and 5?

3. *How many* must be added to 10 to make 11? *To make 12? 13? 14? 15?*

4. One taken from 11 leaves how many? 2 from 12? 3 from 13? 4 from 14? 5 from 15?

5. How many are 10 and 2? 10 and 5? 10 and 3? 10 and 4? 10 and 1?

6. Make on your slate 11, 12, 13, 14, 15.

Eleven.	Twelve.	Thirteen.	Fourteen.	Fifteen.
11.	12.	13.	14.	15.

7. How many books are 10 books and 6 books? 10 books and 7 books? 10 books and 8 books? 10 books and 9 books? 10 books and 10 books?

8. How many are 10 and 6? 10 and 7? 10 and 8? 10 and 9? 10 and 10?

9. How many must be added to 10 to make 16? To make 17? 18? 19? 20?

Sixteen.	Seventeen.	Eighteen.	Nineteen.	Twenty.
16.	17.	18.	19.	20.

10. Six taken from 16 leaves how many? 7 from 17? 8 from 18? 9 from 19? 10 from 20?

11. How many are 10 and 7? 10 and 5? 10 and 4? 10 and 8? 10 and 3? 10 and 9? 10 and 6?

12. Three from 13 leaves how many? 5 from 15? 7 from 17? 9 from 19? 8 from 18? 6 from 16?

13. Make on your slate 16, 17, 18, 19, 20.

14. Copy on slate and read: 11, 13, 15, 17, 19. 18, 16, 20, 14, 12, 10.

LESSON IV.

1. How many balls on each wire of the numeral frame? How many wires has it?

2. How many balls on the two upper wires? How many balls on these two wires and two balls on the third wire counted together?

3. How many balls are 20 balls and 3 balls? 20 balls and 4 balls? 20 balls and 5 balls? 20 balls and 6 balls? 20 balls and 7 balls? 20 balls and 8 balls? 20 balls and 9 balls? 20 balls and 10 balls?

4. How many are 20 and 1? 20 and 3? 20 and 5? 20 and 7? 20 and 9? 20 and 2? 20 and 4? 20 and 6? 20 and 8?

5. Copy on your slate and read: 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

6. How many balls on the three lower wires counted together? How many balls on four wires? On five wires? On 7 wires? On 8 wires? On 6 wires? On 9 wires? On 10 wires?

7. How many balls are 30 balls and 10 balls? 40 balls and 10 balls? 50 balls and 10 balls? 60 balls and 10 balls? 70 balls and 10 balls? 80 balls and 10 balls? 90 balls and 10 balls?

8. How many ones in 2 tens? In 3 tens? 4 tens? 5 tens? 6 tens? 7 tens? 8 tens? 9 tens? 10 tens?

9. How many tens in twenty? In thirty? Forty? *Fifty?* Sixty? Seventy? Eighty? Ninety? One *hundred?*

10. Count from 50 to 60; from 60 to 70.

11. Count from 80 to 90; from 90 to 100.

12. Count from ten to one hundred by tens, thus: Ten, twenty, thirty, forty, fifty, etc.

Copy on slate and read the following numbers:

13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
10	50	90	43	47	56	77	66
20	60	100	44	48	72	88	76
30	70	41	45	49	93	99	86
40	80	42	46	65	39	55	96

LESSON V.

When a number is expressed by two figures, the right-hand figure denotes ones or *units*, and the left-hand figure denotes *tens*.

Thus, in 34 the 3 denotes *tens* and the 4 *units*.

1. Which figure in 36 denotes units? Which figure denotes tens?

2. How many tens and how many units are there in 54? In 63? 75? 87? 49? 93? 56? 77? 66? 78?

Express in figures the following numbers:

3.	**4.**	**5.**	**6.**
Twelve.	Forty-two.	Sixty-six.	Thirty-nine.
Eighteen.	Forty-six.	Seventy-seven.	Forty-nine.
Fifteen.	Sixty-three.	Eighty-eight.	Sixty-nine.
Thirty.	Sixty-nine.	Forty-four.	Twenty-one.
Forty.	Seventy-two.	Fifty-five.	Eighty-seven.
Seventy.	Seventy-four.	Ninety-nine.	Ninety-seven.

7. How many balls on one numeral frame?

8. How many balls on two numeral frames counted together? On three numeral frames?

9. How many balls on 4 numeral frames? On 5 frames? On 6 frames?

10. How many balls on 7 frames? On 8 frames? On 9 frames? On 10 frames?

Copy on slate and read the following numbers:

11.	12.	13.	14.	15.
100	400	700	200	300
200	500	800	400	500
300	600	900	600	700

ADDITION AND SUBTRACTION.

LESSON VI.

ORAL EXERCISES.

1. There [redacted] birds on a tree, and two birds on another [redacted] many birds on both trees? How many [redacted] birds? 2 birds and 3 birds?

2. There are six birds on a limb: if two birds fly away, how many birds will be left? 2 birds from 6 birds leave how many?

3. Mary has written 7 words: if she write 2 words more, how many words will she have written?

4. If Mary write 7 words on her slate, and then erase 2 of them, how many words will be left?

5. How many birds are 5 birds and 2 birds? 8 birds and 2 birds? 7 birds and 2 birds? 9 birds and 2 birds? 10 birds and 2 birds?

6. How many words are 8 words less 2 words? 7 words less 2 words? 10 words less 2 words? 11 words less 2 words? 12 words less 2 words?

To TEACHERS.—Children should not be taught the analysis of these problems. They should add or subtract and give the result.

How many are:		*How many are:*	
1 and 1?	2 less 1?	1 and 2?	3 less 2?
2 and 1?	3 less 1?	2 and 2?	4 less 2?
3 and 1?	4 less 1?	3 and 2?	5 less 2?
4 and 1?	5 less 1?	4 and 2?	6 less 2?
5 and 1?	6 less 1?	5 and 2?	7 less 2?
6 and 1?	7 less 1?	6 and 2?	8 less 2?
7 and 1?	8 less 1?	7 and 2?	9 less 2?
8 and 1?	9 less 1?	8 and 2?	10 less 2?
9 and 1?	10 less 1?	9 and 2?	11 less 2?
10 and 1?	11 less 1?	10 and 2?	12 less 2?

NOTE.—The tables may be recited thus: 1 and 1 are 2; 2 less 1 is 1. 2 and 1 are 3; 3 less 1 is 2, etc.

7. How many balls are 7 balls and 2 balls? 17 balls and 2 balls? 27 balls and 2 balls? 37 balls and 2 balls? 47 balls and 2 balls? 57 balls and 2 balls?

8. How many are 6 and 2? 16 and 2? 26 and 2? 46 and 2? 66 and 2? 86 and 2? 96 and 2?

9. How many is 8 less 2? 18 less 2? 28 less 2? 48 less 2? 68 less 2? 88 less 2? 98 less 2?

10. How many are 8 and 2? 10 less 2? 18 and 2? 20 less 2? 28 and 2? 30 less 2? 48 and 2?

11. How many are 9 and 2? 11 less 2? 29 and 2? 31 less 2? 49 and 2? 51 less 2? 69 and 2?

WRITTEN EXERCISES.

	2	2	2	1	2	2
1. Add	7	9	8	9	6	5
2. From	9	11	10	10	8	7
Take	2	2	2	1	2	2
	2	2	2	2	2	2
3. Add	17	16	26	36	38	29
4. From	19	18	28	38	40	31
Take	2	2	2	2	2	2

	5.	**6.**	**7.**	**8.**	**9.**	**10.**
	1	2	2	2	2	2
	1	2	2	2	1	1
	1	2	2	1	2	2
	1	2	2	2	1	1
	1	2	2	1	2	2
	1	2	2	1	1	2
	1	2	2	2	2	2
	1	2	2	2	1	1
	1	2	2	2	2	1
	1	2	2	1	1	1
	1	2	2	2	2	2
Add	1	2	1	1	1	2

Copy on slate and read the following numbers:

11.	**12.**	**13.**	**14.**	**15.**
101	106	201	206	205
102	107	202	207	207
103	108	203	208	208
104	109	204	209	209
105	110	205	210	203

LESSON VII.

ORAL EXERCISES.

1. A boy has 5 oranges in his hat and 3 in his hands: how many oranges has he?

2. A boy bought 12 oranges, and gave 3 of them away: how many oranges had he left?

3. How many plums are 6 plums and 3 plums? 7 plums and 3 plums?

4. How many plums are 9 plums less 3 plums? 10 plums less 3 plums? 11 plums less 3 plums?

How many are:		***Read and complete:***	
1 and 3?	4 less 3?	$1+3=$	$4-3=$
2 and 3?	5 less 3?	$2+3=$	$5-3=$
3 and 3?	6 less 3?	$3+3=$	$6-3=$
4 and 3?	7 less 3?	$4+3=$	$7-3=$
5 and 3?	8 less 3?	$5+3=$	$8-3=$
6 and 3?	9 less 3?	$6+3=$	$9-3=$
7 and 3?	10 less 3?	$7+3=$	$10-3=$
8 and 3?	11 less 3?	$8+3=$	$11-3=$
9 and 3?	12 less 3?	$9+3=$	$12-3=$
10 and 3?	13 less 3?	$10+3=$	$13-3=$

NOTE.—The teacher should make these tables on the board, and, after teaching the signs +, —, and =, drill the pupils until they can read and complete with rapidity.

5. How many are 6 and 3? 9 less 3? 10 less 3? 13 less 3? 12 less 3? 11 less 3?

6. How many are 7 and 3? 10 less 3? 9 and 3? 12 less 3? 8 and 3? 11 less 3? 10 and 3? 13 less 3? 11 and 3? 14 less 3?

7. How many pens are 6 pens and 3 pens? 16 pens and 3 pens? 26 pens and 3 pens? 46 pens and 3 pens? 66 pens and 3 pens? 86 pens and 3 pens? 96 pens and 3 pens?

8. How many are 7 and 3? 17 and 3? 27 and 3? 47 and 3? 57 and 3? 67 and 3? 87 and 3?

9. How many is 10 less 3? 30 less 3? 40 less 3? 60 less 3? 50 less 3? 80 less 3? 90 less 3?

10. How many are 5 and 3? 8 less 3? 15 and 3? 18 less 3? 35 and 3? 38 less 3?

11. How many are 8 and 3? 11 less 3? 18 and 3? 21 less 3? 28 and 3? 31 less 3? 48 and 3? 51 less 3? 68 and 3? 71 less 3?

12. How many are 9 and 3? 12 less 3? 29 and 3? 32 less 3? 49 and 3? 52 less 3? 69 and 3? 72 less 3? 89 and 3? 92 less 3?

WRITTEN EXERCISES.

	3	3	3	3	3
1. Add	8	9	12	13	15
2. From	11	12	15	16	18
Take	3	3	3	3	3
	13	23	33	23	33
3. Add	18	28	48	37	57
4. From	21	31	51	40	60
Take	3	3	3	3	3

	5.	6.	7.	8.	9.	10.
	3	3	3	3	3	3
	3	3	3	3	3	2
	3	3	3	1	3	2
	3	3	3	1	2	1
	3	3	3	3	2	1
	3	3	3	1	3	2
	3	3	3	3	2	3
	3	3	3	1	3	3
	3	3	3	3	3	2
	3	3	3	1	2	1
	3	3	3	1	2	1
	3	3	3	1	3	2
Add	3	2	1	3	3	1

Copy on slate and read the following numbers:

11.	12.	13.	14.	15.	16.
110	115	212	311	412	208
111	116	214	315	405	305
112	117	216	317	415	404
113	118	218	313	406	606
114	119	215	319	416	707

LESSON VIII.

ORAL EXERCISES.

1. A farmer has 7 sheep in one field and 4 sheep in another: how many sheep are in both fields?

2. A farmer has 13 hogs, and sells 4 of them: how many hogs has he left?

3. There are 8 pigs in one pen and 4 pigs in another pen: how many pigs are there in both of the pens?

4. There are 14 pupils in a class: if 4 of them leave the class, how many pupils will remain?

5. How many pears are 7 pears and 4 pears? 9 pears and 4 pears? 10 pears and 4 pears? 8 pears and 4 pears? 6 pears and 4 pears?

6. How many lines are 11 lines less 4 lines? 12 lines less 4 lines? 10 lines less 4 lines? 13 lines less 4 lines? 14 lines less 4 lines?

7. How many are 7 and 4? 11 less 4? 9 and 4? 13 less 4? 8 and 4? 12 less 4? 10 and 4? 14 less 4? 6 and 4? 10 less 4?

8. How many are 11 less 4? 13 less 4? 12 less 4? 9 less 4? 14 less 4? 10 less 4?

How many are:		***Read and complete:***	
1 and 4?	5 less 4?	$1+4=$	$5-4=$
2 and 4?	6 less 4?	$2+4=$	$6-4=$
3 and 4?	7 less 4?	$3+4=$	$7-4=$
4 and 4?	8 less 4?	$4+4=$	$8-4=$
5 and 4?	9 less 4?	$5+4=$	$9-4=$
6 and 4?	10 less 4?	$6+4=$	$10-4=$
7 and 4?	11 less 4?	$7+4=$	$11-4=$
8 and 4?	12 less 4?	$8+4=$	$12-4=$
9 and 4?	13 less 4?	$9+4=$	$13-4=$
10 and 4?	14 less 4?	$10+4=$	$14-4=$

9. How many dimes are 6 dimes and 4 dimes? 16 dimes and 4 dimes? 36 dimes and 4 dimes? 56 dimes and 4 dimes? 46 dimes and 4 dimes? 66 dimes and 4 dimes?

10. How many are 7 and 4? 17 and 4? 37 and 4? 57 and 4? 27 and 4? 47 and 4? 67 and 4? 87 and 4?

11. How many are 9 and 4? 19 and 4? 39 and 4? *29 and* 4? 49 and 4? 59 and 4? 69 and 4? 89 *and 4?*

12. How many are 8 and 4? 18 and 4? 48 and 4? 38 and 4? 28 and 4? 58 and 4? 68 and 4? 88 and 4? 98 and 4? 78 and 4?

13. How many is 12 less 4? 22 less 4? 32 less 4? 52 less 4? 42 less 4? 62 less 4? 72 less 4?

14. How many is 13 less 4? 33 less 4? 43 less 4? 63 less 4? 73 less 4? 53 less 4? 83 less 4?

15. How many are 16 and 4? 20 less 4? 46 and 4? 50 less 4? 27 and 4? 31 less 4? 77 and 4? 81 less 4? 58 and 4? 62 less 4? 68 and 4? 72 less 4?

WRITTEN EXERCISES.

	4	4	4	4	4
1. Add	13	9	11	12	15
2. From	17	13	15	16	19
Take	4	4	4	4	4
	24	14	14	14	34
3. Add	27	37	39	56	26
4. From	46	56	58	75	66
Take	14	14	14	14	34

	5.	**6.**	**7.**	**8.**	**9.**	**10.**
	4	4	4	4	3	4
	4	4	4	4	4	3
	4	4	4	2	3	2
	4	4	4	1	3	4
	4	4	4	3	4	1
	4	4	4	4	3	4
	4	4	4	2	4	4
	4	4	4	3	4	4
	4	4	4	4	4	3
	4	4	4	1	3	3
Add	4	3	2	4	3	4

Copy on slate and read the following numbers:

11.	**12.**	**13.**	**14.**	**15.**	**16.**
100	101	600	606	111	425
200	202	700	707	222	625
300	303	800	808	333	825
400	404	900	909	444	725
500	505	222	333	555	925
600	306	333	409	675	874

LESSON IX.

ORAL EXERCISES.

1. A boy has 4 nuts in one hand and 5 nuts in the other: how many nuts in both his hands?

2. Harry had 12 nuts, and gave 5 of them to Mary: how many nuts had he left?

3. There are 9 cows in one pasture and 5 cows in another; how many cows in both pastures?

4. How many are 9 cows less 5 cows? 12 cows less 5 cows? 10 cows less 5 cows?

5. How many are 6 and 5? 4 and 5? 7 and 5? 8 and 5? 9 and 5? 10 and 5?

6. How many is 6 less 5? 8 less 5? 10 less 5? 12 less 5? 14 less 5?

7. How many is 7 less 5? 9 less 5? 11 less 5? 13 less 5? 15 less 5?

8. How many are 8 and 5? 13 less 5? 10 and 5? 15 less 5? 7 and 5? 12 less 5? 9 and 5? 14 less 5?

9. How many are 11 less 5? 16 less 5? 18 less 5? *20 less 5? 17 less 5?* 13 less 5? 12 less 5?

How many are:		*Read and complete:*	
1 and 5?	6 less 5?	1 + 5 =	6 − 5 =
2 and 5?	7 less 5?	2 + 5 =	7 − 5 =
3 and 5?	8 less 5?	3 + 5 =	8 − 5 =
4 and 5?	9 less 5?	4 + 5 =	9 − 5 =
5 and 5?	10 less 5?	5 + 5 =	10 − 5 =
6 and 5?	11 less 5?	6 + 5 =	11 − 5 =
7 and 5?	12 less 5?	7 + 5 =	12 − 5 =
8 and 5?	13 less 5?	8 + 5 =	13 − 5 =
9 and 5?	14 less 5?	9 + 5 =	14 − 5 =
10 and 5?	15 less 5?	10 + 5 =	15 − 5 =

10. How many cents are 6 cents and 5 cents? 16 cents and 5 cents? 36 cents and 5 cents? 56 cents and 5 cents? 76 cents and 5 cents?

11. How many are 11 cents less 5 cents? 21 cents less 5 cents? 41 cents less 5 cents? 61 cents less 5 cents? 81 cents less 5 cents?

12. How many are 7 and 5? 27 and 5? 47 and 5? 67 and 5? 87 and 5? 77 and 5?

13. How many are 5 and 5? 15 and 5? 25 and 5? 45 and 5? 65 and 5? 85 and 5? 75 and 5?

14. How many are 8 and 5? 28 and 5? 48 and 5? 68 and 5? 88 and 5? 58 and 5?

15. How many are 9 and 5? 19 and 5? 39 and 5? 29 and 5? 49 and 5? 69 and 5? 59 and 5?

16. How many is 12 less 5? 22 less 5? 32 less 5? 42 less 5? 52 less 5? 62 less 5? 72 less 5?

17. How many is 13 less 5? 33 less 5? 53 less 5? 63 less 5? 83 less 5? 73 less 5?

18. How many is 14 less 5? 24 less 5? 34 less 5? 54 less 5? 44 less 5? 64 less 5?

19. How many is 11 less 5? 21 less 5? 41 less 5? 61 less 5? 51 less 5? 31 less 5? 71 less 5?

20. How many is 20 less 5? 40 less 5? 60 less 5? 50 less 5? 70 less 5? 80 less 5?

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
	5	5	5	1	5	5	4	4	4
	5	5	5	1	5	2	4	3	2
	5	5	5	5	2	1	4	4	3
	5	5	5	5	2	2	4	3	4
	5	5	5	5	5	5	4	4	1
	5	5	5	1	5	3	4	3	3
	5	5	5	5	5	5	4	4	2
	5	5	5	1	2	5	4	3	1
	5	5	5	1	5	3	4	4	2
	5	5	5	5	2	2	4	3	3
	5	5	5	5	2	1	4	4	4
	5	5	5	1	5	2	4	3	3
	5	5	5	1	2	3	4	4	2
Add	5	1	3	5	2	5	3	3	1

	10.	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**
	15	15	25	25	35	35	45	45
Add	16	27	28	39	29	36	26	25

	18.	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**
From	15	14	24	22	32	41	61	53
Take	5	5	15	15	15	25	35	25

Copy and read the following numbers:

26.	**27.**	**28.**	**29.**	**30.**
122	223	234	244	220
217	227	239	246	230
118	228	236	248	240
216	229	235	245	250
415	226	237	247	260
518	*225*	238	249	270

LESSON X.

ORAL EXERCISES.

1. Frank caught 8 trout, and James 6 trout: how many trout did both catch?

2. A boy caught 13 fishes, and gave 6 of them to a poor boy: how many fishes had he left?

3. Charles gave 9 cents for pencils and 6 cents for paper: how many cents did he spend?

4. A boy wrote 15 words on his slate, and then erased 6 of them: how many words were left on his slate?

5. How many are 4 and 6? 6 and 6? 5 and 6? 7 and 6? 9 and 6? 8 and 6? 10 and 6?

6. How many is 8 less 6? 10 less 6? 9 less 6? 11 less 6? 12 less 6? 13 less 6? 14 less 6? 15 less 6? 16 less 6?

How many are:		*Read and complete:*	
1 and 6?	7 less 6?	$1+6=$	$7-6=$
2 and 6?	8 less 6?	$2+6=$	$8-6=$
3 and 6?	9 less 6?	$3+6=$	$9-6=$
4 and 6?	10 less 6?	$4+6=$	$10-6=$
5 and 6?	11 less 6?	$5+6=$	$11-6=$
6 and 6?	12 less 6?	$6+6=$	$12-6=$
7 and 6?	13 less 6?	$7+6=$	$13-6=$
8 and 6?	14 less 6?	$8+6=$	$14-6=$
9 and 6?	15 less 6?	$9+6=$	$15-6=$
10 and 6?	16 less 6?	$10+6=$	$16-6=$

7. How many dimes are 5 dimes and 6 dimes? 25 dimes and 6 dimes? 45 dimes and 6 dimes? 65 dimes and 6 dimes? 85 dimes and 6 dimes?

8. How many are 12 cents less 6 cents? 32 cents less 6 cents? 52 cents less 6 cents? 72 cents less 6 cents? 92 cents less 6 cents?

9. How many are 7 and 6? 27 and 6? 37 and 6? 57 and 6? 67 and 6? 87 and 6? 77 and 6?

10. How many are 8 and 6? 18 and 6? 28 and 6? 38 and 6? 48 and 6? 58 and 6? 68 and 6? 78 and 6? 88 and 6?

11. How many are 9 and 6? 29 and 6? 39 and 6? 59 and 6? 79 and 6? 69 and 6? 89 and 6?

12. How many is 16 less 6? 26 less 6? 36 less 6? 46 less 6? 66 less 6? 76 less 6? 86 less 6?

13. How many is 9 less 6? 11 less 6? 13 less 6? 14 less 6? 10 less 6? 15 less 6? 16 less 6?

14. How many is 8 less 6? 10 less 5? 13 less 5? 12 less 4? 13 less 4? 15 less 6? 14 less 6?

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**
	6	6	6	6	6	4	6	3	4	6	6	6
	6	6	6	6	3	6	4	4	3	5	5	5
	6	6	6	3	6	4	3	6	4	4	1	3
	6	6	6	3	3	6	4	6	2	4	5	3
	6	6	6	3	2	4	6	5	1	4	6	4
	6	6	2	6	3	6	4	6	2	4	1	5
	6	6	1	3	6	4	6	5	3	4	2	6
	6	6	6	6	2	4	1	6	4	5	3	4
	6	6	6	6	3	4	3	6	4	5	5	2
	6	6	6	6	6	4	4	5	4	5	6	1
	6	6	6	6	2	6	6	5	4	5	1	6
	6	6	6	6	3	6	6	6	3	6	2	5
	6	6	6	6	1	6	4	6	3	6	3	3
	6	6	6	6	2	6	3	6	3	6	4	4
	6	6	6	6	3	6	2	6	3	6	5	6
Add	*6*	*1*	*2*	*3*	*6*	*4*	*1*	*5*	*3*	*6*	*6*	*5*

	13.	14.	15.	16.	17.	18.
	16	46	24	56	63	66
	26	23	44	46	64	55
	22	43	36	36	46	44
	23	16	35	61	36	33
Add	36	36	46	66	25	22

	19.	20.	21.	22.	23.	24.
From	13	48	69	88	99	43
Take	6	26	36	46	66	16

Copy and read the following numbers:

25.	26.	27.	28.	29.
324	404	414	427	434
336	507	419	437	444
316	608	413	447	454
344	709	411	457	464
355	803	410	467	474

LESSON XI.

ORAL EXERCISES.

1. Harry found 8 eggs in one nest and 7 eggs in another: how many eggs did he find?

2. Charles picked 9 quarts of cherries from one tree and 7 quarts from another tree: how many quarts of cherries did he pick?

3. Susan picked 12 nice peaches, and gave her mother 7 of them: how many had Susan left?

4. A man, having 13 sheep, sold 7 of them: how many sheep had he left?

How many are:		*Read and complete:*	
1 and 7?	8 less 7?	1 + 7 =	8 − 7 =
2 and 7?	9 less 7?	2 + 7 =	9 − 7 =
3 and 7?	10 less 7?	3 + 7 =	10 − 7 =
4 and 7?	11 less 7?	4 + 7 =	11 − 7 =
5 and 7?	12 less 7?	5 + 7 =	12 − 7 =
6 and 7?	13 less 7?	6 + 7 =	13 − 7 =
7 and 7?	14 less 7?	7 + 7 =	14 − 7 =
8 and 7?	15 less 7?	7 + 7 =	15 − 7 =
9 and 7?	16 less 7?	9 + 7 =	16 − 7 =
10 and 7?	17 less 7?	10 + 7 =	17 − 7 =

5. How many are 2 and 7? 4 and 7? 6 and 7? 8 and 7? 10 and 7? 3 and 7?

6. How many are 5 and 7? 7 and 7? 9 and 7? 6 and 7? 10 and 7? 8 and 7?

7. How many is 7 less 6? 8 less 7? 10 less 6? 9 less 7? 11 less 7? 13 less 7? 12 less 7? 14 less 7? 15 less 7? 17 less 7? 16 less 7? 18 less 7?

8. How many quarts are 6 quarts and 7 quarts? 16 quarts and 7 quarts? 36 quarts and 7 quarts? 46 quarts and 7 quarts? 66 quarts and 7 quarts?

9. How many are 13 pints less 7 pints? 27 pints less 7 pints? 17 pints less 7 pints? 47 pints less 7 pints? 57 pints less 7 pints?

10. How many are 5 and 7? 15 and 7? 27 and 7? 47 and 7? 67 and 7? 87 and 7?

11. How many are 8 and 7? 28 and 7? 38 and 7? 48 and 7? 68 and 7? 88 and 7? 78 and 7?

12. How many are 9 and 7? 19 and 7? 29 and 7? 39 and 7? 59 and 7? 69 and 7? 89 and 7?

13. How many is 9 less 7? 11 less 7? 10 less 7? 12 less 7? 14 less 7? 13 less 7? 16 less 7? 15 less 7? 17 less 7?

14. *How* many is 13 less 4? 13 less 5? 13 less 6? *13 less 7? 14 less 6?* 14 less 5? 14 less 7?

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**
	7	7	7	7	7	7	7	1	7	7	6	7
	7	7	7	7	7	7	2	2	6	5	5	6
	7	7	7	7	7	7	7	3	7	7	7	5
	7	7	7	7	7	7	2	4	5	5	5	5
	7	7	7	7	7	7	1	5	7	7	6	4
	7	7	7	7	7	7	3	6	3	5	6	3
	7	7	7	7	7	7	3	5	7	7	7	2
	7	7	7	7	7	7	7	4	2	5	5	2
	7	7	7	7	7	7	2	3	7	7	6	1
	7	7	7	7	7	7	7	2	1	5	7	1
Add	7	1	2	3	4	5	3	1	7	7	6	7

	13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**
	12	17	34	21	37	27	50
	13	27	24	37	44	55	17
	14	37	64	60	77	37	63
	15	47	54	42	22	43	57
Add	16	67	74	67	33	65	77

	20.	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**
From	21	31	45	66	83	60	50
Take	17	17	27	37	57	37	27

Copy and read the following numbers:

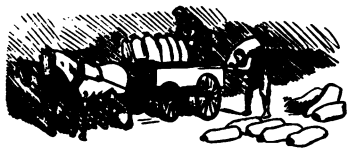
27.	**28.**	**29.**	**30.**	**31.**	**32.**
210	204	325	446	$600	$700
320	405	426	660	$603	$703
630	670	531	550	$630	$730
740	730	631	505	$633	$744
840	408	342	515	$666	$755

NOTE.—The character $ denotes dollars, and is called the *dollar sign;* $210 is read 210 *dollars;* $446 is read 446 dollars.

LESSON XII.

ORAL EXERCISES.

1. Mary has written 7 words in one line and 8 words in another line: how many words in both lines?



2. A farmer put 6 sacks of flour in his wagon, and the miller put in 8 sacks more: how many sacks of flour in the wagon?

3. A wagon contains 15 bags of wheat: if 8 bags be taken out, how many bags will be left?

4. Charles earned \$16, and paid \$8 for a coat: how many dollars had he left?

5. Kate has 9 plums in her right hand and 8 plums in her left hand: how many plums in both her hands?

6. How many are 2 cents and 8 cents? 3 cents and 8 cents? 4 cents and 8 cents? 6 cents and 8 cents? 7 cents and 8 cents?

7. How many are 3 and 8? 5 and 8? 7 and 8? 9 and 8? 10 and 8? 6 and 8? 8 and 8?

8. How many is 10 less 8? 12 less 8? 14 less 8? 16 less 8? 18 less 8? 13 less 8? 15 less 8? 17 less 8?

How many are:		*Read and complete:*	
1 and 8?	9 less 8?	$1+8=$	$9-8=$
2 and 8?	10 less 8?	$2+8=$	$10-8=$
3 and 8?	11 less 8?	$3+8=$	$11-8=$
4 and 8?	12 less 8?	$4+8=$	$12-8=$
5 and 8?	13 less 8?	$5+8=$	$13-8=$
6 and 8?	14 less 8?	$6+8=$	$14-8=$
7 and 8?	15 less 8?	$7+8=$	$15-8=$
8 and 8?	16 less 8?	$8+8=$	$16-8=$
9 and 8?	17 less 8?	$9+8=$	$17-8=$
10 and 8?	*18 less 8?*	$10+8=$	$18-8=$

9. How many pecks are 4 pecks and 8 pecks? 14 pecks and 8 pecks? 24 pecks and 8 pecks? 44 pecks and 8 pecks? 64 pecks and 8 pecks?

10. How many sacks are 5 sacks and 8 sacks? 25 sacks and 8 sacks? 55 sacks and 8 sacks? 75 sacks and 8 sacks? 85 sacks and 8 sacks?

11. How many are 7 and 8? 27 and 8? 47 and 8? 67 and 8? 87 and 8? 77 and 8? 97 and 8?

12. How many are 8 and 8? 28 and 8? 38 and 8? 58 and 8? 48 and 8? 68 and 8? 78 and 8?

13. How many are 9 and 8? 19 and 8? 29 and 8? 39 and 8? 49 and 8? 59 and 8? 69 and 8? 89 and 8? 99 and 8?

14. How many is 12 less 8? 14 less 8? 16 less 8? 26 less 8? 36 less 8? 15 less 8? 25 less 8?

15. Begin with 5 and add to 100 by fives, thus: 5, 10, 15, 20, 25, 30, etc.

16. Begin with 3 and add to 63 by fives.

17. Begin with 2 and add to 72 by fives.

WRITTEN EXERCISES.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
8	8	8	8	8	8	8	8	8	8	8	8
8	8	8	8	8	8	3	5	6	7	7	6
8	8	8	8	8	8	8	8	6	8	6	6
8	8	8	8	8	8	3	5	8	7	5	7
8	8	8	8	8	8	8	8	6	8	4	7
8	8	8	8	8	8	4	5	8	7	3	5
8	8	8	8	8	8	3	5	6	8	2	5
8	8	8	8	8	8	4	5	6	7	3	8
8	8	8	8	8	8	8	8	8	8	4	8
8	8	8	8	8	8	8	8	6	7	5	4
8	8	8	8	8	8	4	5	8	8	6	4
8	8	8	8	8	8	3	8	6	7	7	8
Add 8	1	2	3	4	5	2	8	8	8	8	8

	13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**
	18	28	41	23	44	25	$36
	28	32	60	33	55	70	$47
	38	46	78	43	60	32	$58
	48	38	81	53	70	68	$63
	58	28	38	63	66	47	$77
	68	16	46	78	77	88	$88
Add	78	50	88	88	88	55	$34

	20.	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**
From	$45	55	75	$65	36	46	$56
Take	$18	38	58	$28	18	28	$48

27. Add 37, 28, 49, 50, 61, 37, 82, 43, and 54.

28. Add 63, 38, 45, 54, 73, 47, 88, 66, and 78.

29. Add $61, $62, $63, $64, $65, $66, $67, and $68.

LESSON XIII.

ORAL EXERCISES.

1. A school-house has 5 windows in the ends and 9 windows in the sides: how many windows in the house?

2. Mary solved 7 problems on her slate and 9 other problems on paper: how many problems did she solve?

3. Albert gave 8 cents for a slate and 9 cents for paper: how many cents did he pay for both?

4. A farmer gave 6 dollars for a yoke and chain, and 9 dollars for a plow: how many dollars did they cost?

5. Harry picked 13 peaches, and gave 9 of them to *his playmates:* how many had he left?

6. How many are 1 and 9? 2 and 9? 3 and 9? 4 and 9? 5 and 9? 6 and 9? 7 and 9? 8 and 9? 9 and 9? 10 and 9?

How many are:		*Read and complete;*	
1 and 9?	10 less 9?	1 + 9 =	10 − 9 =
2 and 9?	11 less 9?	2 + 9 =	11 − 9 =
3 and 9?	12 less 9?	3 + 9 =	12 − 9 =
4 and 9?	13 less 9?	4 + 9 =	13 − 9 =
5 and 9?	14 less 9?	5 + 9 =	14 − 9 =
6 and 9?	15 less 9?	6 + 9 =	15 − 9 =
7 and 9?	16 less 9?	7 + 9 =	16 − 9 =
8 and 9?	17 less 9?	8 + 9 =	17 − 9 =
9 and 9?	18 less 9?	9 + 9 =	18 − 9 =
10 and 9?	19 less 9?	10 + 9 =	19 − 9 =

7. How many is 10 less 9? 12 less 9? 14 less 9? 16 less 9? 18 less 9? 20 less 9?

8. How many is 11 less 9? 13 less 9? 15 less 9? 17 less 9? 19 less 9? 18 less 9? 16 less 9?

9. How many men are 7 men and 9 men? 27 men and 9 men? 37 men and 9 men? 47 men and 9 men? 67 men and 9 men?

10. How many hours are 18 hours and 9 hours? 28 hours and 9 hours? 38 hours and 9 hours? 58 hours and 9 hours? 68 hours and 9 hours?

11. How many are 6 and 9? 26 and 9? 46 and 9? 66 and 9? 86 and 9? 76 and 9? 56 and 9?

12. How many are 15 and 9? 25 and 9? 35 and 9? 85 and 9? 75 and 9? 65 and 9? 55 and 9? 45 and 9?

13. How many is 15 less 9? 25 less 9? 35 less 9? 45 less 9? 55 less 9? 65 less 9? 75 less 9?

14. Begin with 1 and add to 61 by fours.

15. Begin with 2 and add to 62 by threes.

16. Begin with 3 and add to 63 by fours.

WRITTEN EXERCISES.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
	9	9	9	9	9	9	9	9	9	9	9	9
	9	9	9	9	9	9	4	5	9	8	8	6
	9	9	9	9	9	9	9	5	6	7	7	9
	9	9	9	9	9	9	4	9	6	6	9	7
	9	9	9	9	9	9	9	9	9	5	3	9
	9	9	9	9	9	9	4	5	9	4	9	8
	9	9	9	9	9	9	9	5	6	9	4	9
	9	9	9	9	9	9	4	9	6	4	5	9
	9	9	9	9	9	9	9	9	9	5	6	9
	9	9	9	9	9	9	4	5	9	6	8	3
	9	9	9	9	9	9	9	5	4	7	9	3
Add	9	3	5	7	6	8	4	9	6	8	8	9

	13.	14.	15.	16.	17.	18.	19.
	46	39	29	69	70	118	139
	64	93	92	96	89	129	184
	73	49	89	79	60	207	109
	68	94	98	97	99	209	208
	47	59	79	86	40	127	169
Add	59	95	97	68	59	186	155

	20.	21.	22.	23.	24.	25.	26.
From	46	47	58	346	574	666	555
Take	29	19	39	129	249	349	238

	27.	28.	29.	30.	31.	32.	33.
From	50	70	90	293	438	529	607
Take	19	49	69	159	293	395	394

34. Add 69, 79, 89, 99, 76, 86, 56, 46, 77.

35. Add 19, 29, 36, 49, 56, 59, 64, 69, 99.

36. *Add $28, $49,* $69, $78, $45, $79, $62, $39.

LESSON XIV.

ORAL EXERCISES.

1. How many are 7 and 6, less 8? 7 and 8, less 6?
2. How many are 5 and 9, less 7? 6 and 10, less 9?
3. How many are 7 and 8, less 9? 6 and 8, less 7?
4. How many are 7 and 7, less 8? 7 and 6, less 9?
5. How many are 8 and 6, less 7? 8 and 5, less 8?
6. How many are 9 and 6, less 8? 7 and 6, less 10?
7. How many are 5 and 9, less 8? 7 and 8, less 6?
8. How many are 7 and 5, less 8? 9 and 5, less 8?
9. How many are 7+5+6+6—8? 8+4+6+7—6?
10. How many are 6+9+8+7—9? 5+7+8+9—10?

How many are:		*Read and complete:*	
1 and 10?	11 less 10?	1 + 10 =	11 — 10 =
2 and 10?	12 less 10?	2 + 10 =	12 — 10 =
3 and 10?	13 less 10?	3 + 10 =	13 — 10 =
4 and 10?	14 less 10?	4 + 10 =	14 — 10 =
5 and 10?	15 less 10?	5 + 10 =	15 — 10 =
6 and 10?	19 less 10?	6 + 10 =	16 — 10 =
7 and 10?	17 less 10?	7 + 10 =	17 — 10 =
8 and 10?	18 less 10?	8 + 10 =	18 — 10 =
9 and 10?	19 less 10?	9 + 10 =	19 — 10 =
10 and 10?	20 less 10?	10 + 10 =	20 — 10 =

11. Begin with 3 and add to 63 by 6's.
12. Begin with 7 and and to 70 by 7's.
13. Begin with 3 and add to 73 by 7's.
14. Begin with 4 and add to 64 by 6's.
15. Begin with 50 and subtract by 2's to 0.
16. Begin with 40 and subtract by 3's to 1.
17. Begin with 60 and subtract by 5's to 0.
18. Begin with 50 and subtract by 4's to 2.
19. Begin with 100 and subtract by 10's to 0.

20. Add 6, 7, 5, 4, 9, 3, 10, 7, 6, 3, 5, 7, 8.
21. Add 17, 8, 9, 3, 10, 7, 9, 8, 4, 5, 8, 6, 7.
22. Add 37, 6, 7, 8, 9, 4, 3, 10, 7, 6, 3, 4, 9.
23. Add 45, 6, 9, 4, 8, 7, 6, 10, 8, 7, 5, 10, 6.
24. Add 50, 7, 6, 5, 8, 3, 9, 7, 8, 6, 9, 7, 10.
25. Add 10, 5, 7, 8, 9, 3, 4, 5, 6, 7, 8, 6, 9.
26. Add 23, 7, 9, 8, 6, 4, 2, 5, 7, 9, 8, 10.

WRITTEN EXERCISES.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	125	108	135	98	223	463	148
	248	209	146	99	339	78	229
	105	176	178	88	107	139	106
	223	184	199	77	88	65	157
	109	119	138	143	67	173	189
Add	108	104	147	208	78	89	105

	8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**
From	145	247	365	473	586	693	287	364
Take	36	143	246	344	277	475	179	246

	16.	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**
From	428	437	549	558	637	866	688	329
Take	253	184	276	394	275	493	392	187

	24.	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**	**31.**
From	$506	703	$804	706	360	$530	680	770
Take	$342	451	$562	392	183	$275	453	537

	32.	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**
From	$677	$704	815	606	702	731	609	718
Take	*$448*	*$565*	478	470	482	628	477	583

MULTIPLICATION AND DIVISION.

LESSON XV.

ORAL EXERCISES.

1. How many times do I say *la*, children: *la, la?* How many times now: *la, la, la?*

2. Speak *la* twice. Speak *la* three times. Four times.

3. Clap your hands once. Clap them twice. Clap them 4 times. Clap them 5 times.

4. How many are 3 times 1 pencil? 5 times 1 pencil? 4 times 1 pencil? 8 times 1 pencil? 10 times 1 pencil? 6 times 1 pencil?

5. How many are twice 1? 3 times 1? 4 times 1? 5 times 1? 6 times 1?

6. How many are 6 times 1? 7 times 1? 8 times 1? 9 times 1? 10 times 1?

1	1 time 1?	1 in 1?
1+1	2 times 1?	1 in 2?
1+1+1	3 times 1?	1 in 3?
1+1+1+1	4 times 1?	1 in 4?
1+1+1+1+1	5 times 1?	1 in 5?
1+1+1+1+1+1	6 times 1?	1 in 6?
1+1+1+1+1+1+1	7 times 1?	1 in 7?
1+1+1+1+1+1+1+1 . .	8 times 1?	1 in 8?
1+1+1+1+1+1+1+1+1 .	9 times 1?	1 in 9?
1+1+1+1+1+1+1+1+1+1	10 times 1?	1 in 10?

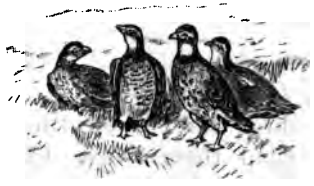
NOTE.—See page 40 for methods of using these tables.

7. How many are 3 times 1? 5 times 1? 7 times 1? 9 times 1? 8 times 1? 6 times 1? 10 times 1?

8. How many times 1 in 3? 1 in 5? 1 in 7? 1 in 9? 1 in 4? 1 in 6? 1 in 8? 1 in 10?

LESSON XVI.

ORAL EXERCISES.



1. How many quails are 2 times 2 quails? 3 times 2 quails? 4 times 2 quails?

2. How many quails are 5 times 2 quails? 6 times 2 quails? 7 times 2 quails?

3. How many keys are 3 times 2 keys? 4 times 2 keys? 5 times 2 keys? 6 times 2 keys? 7 times 2 keys? 8 times 2 keys?

4. How many times 2 keys in 4 keys? 2 keys in 6 keys? 2 keys in 8 keys? 2 keys in 10 keys?

5. How many are 2 times 2? 4 times 2? 6 times 2? 8 times 2? 10 times 2? Twice 2?

2	1 time 2?	2 in 2?
2+2	2 times 2?	2 in 4?
2+2+2	3 times 2?	2 in 6?
2+2+2+2	4 times 2?	2 in 8?
2+2+2+2+2	5 times 2?	2 in 10?
2+2+2+2+2+2	6 times 2?	2 in 12?
2+2+2+2+2+2+2	7 times 2?	2 in 14?
2+2+2+2+2+2+2+2 . . .	8 times 2?	2 in 16?
2+2+2+2+2+2+2+2+2 .	9 times 2?	2 in 18?
2+2+2+2+2+2+2+2+2+2	10 times 2?	2 in 20?

NOTE.—The above tables may be recited in several ways, as follows (taking third line for example): (1) $2+2+2=6$; 3 times $2=6$; 3 in 6 two times. (2) $2+2+2=3$ times $2=6$; 3 in 6 two times. (3) $2+2+2$, or 3 times $2,=6$; 3 in 6 two times. When pupils are sufficiently familiar with the finding of the product by addition, they should then recite the tables of multiplication and division together, thus: 3 times $2=6$; 3 in 6 two times; and finally they should recite the two tables *separately.*

6. How many are 3 times 2? 5 times 2? 7 times 2? 9 times 2? Once 2? 6 times 2? 8 times 2?

7. What is the cost of 6 two-cent postage stamps? 10 two-cent stamps? 8 two-cent stamps?

8. How many times 2 in 2? 2 in 4? 2 in 6? 2 in 8? 2 in 12? 2 in 6? 2 in 10? 2 in 14? 2 in 18?

WRITTEN EXERCISES.

Copy and fill:

$1 \times 2 = 2$	$2 \div 2 = 1$	$3 \times 2 =$
$2 \times 2 =$	$4 \div 2 =$	$6 \div 2 =$
$3 \times 2 =$	$6 \div 2 =$	$5 \times 2 =$
$4 \times 2 =$	$8 \div 2 =$	$10 \div 2 =$
$5 \times 2 =$	$10 \div 2 =$	$7 \times 2 =$
$6 \times 2 =$	$12 \div 2 =$	$14 \div 2 =$
$7 \times 2 =$	$14 \div 2 =$	$8 \times 2 =$
$8 \times 2 =$	$16 \div 2 =$	$16 \div 2 =$
$9 \times 2 =$	$18 \div 2 =$	$9 \times 2 =$
$10 \times 2 =$	$20 \div 2 =$	$18 \div 2 =$

NOTE.—The sign $\times$ may be read *times;* thus, 3×2 may be read 3 times 2.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	3	5	7	8	4	9	10
Multiply	2	2	2	2	2	2	2
	6						

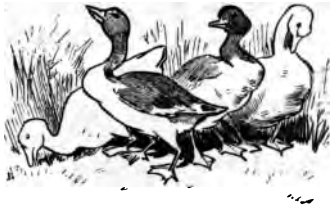
	8.	**9.**	**10.**	**11.**	**12.**	**13.**
Divide	2)10	2)14	2)16	2)8	2)18	2)20

	14.	**15.**	**16.**	**17.**	**18.**	**19.**
	12	22	22	22	222	222
Multiply	2	2	3	4	3	4
	24	44			666	

	20.	**21.**	**22.**	**23.**	**24.**
Divide	2)44	2)66	2)88	2)666	2)888

LESSON XVII.

ORAL EXERCISES.



1. How many geese are twice 2 geese? 3 times 2 geese?

2. How many are 4 times 2 geese? 5 times 2 geese? 6 times 2 geese? 7 times 2 geese?

3. How many books are 3 times 3 books? 4 times 3 books? 6 times 3 books? 8 times 3 books? 9 times 3 books? 10 times 3 books?

4. How many times 3 books in 6 books? 3 books in 9 books? 3 books in 12 books? 3 books in 15 books?

5. How many times 3 books in 18 books? 3 books in 21 books? 3 books in 24 books? 3 books in 27 books? 3 books in 30 books?

3	1 time 3?	3 in 3?
3+3	2 times 3?	3 in 6?
3+3+3	3 times 3?	3 in 9?
3+3+3+3	4 times 3?	3 in 12?
3+3+3+3+3	5 times 3?	3 in 15?
3+3+3+3+3+3	6 times 3?	3 in 18?
3+3+3+3+3+3+3	7 times 3?	3 in 21?
3+3+3+3+3+3+3+3 . .	8 times 3?	3 in 24?
3+3+3+3+3+3+3+3+3 .	9 times 3?	3 in 27?
3+3+3+3+3+3+3+3+3+3	10 times 3?	3 in 30?

6. How many are 3 times 3? 4 times 3? 5 times 3? 6 times 3? 7 times 3? 8 times 3? 9 times 3? 10 times 3?

7. How many are 3 times 3? 5 times 3? 7 times 3? 9 times 3? Once 3? 6 times 3?

8. How many are twice 3? 4 times 3? 6 times 3? 8 times 3? 10 times 3?

9. How many times 3 in 6? 3 in 9? 3 in 15? 3 in 21? 3 in 24? 3 in 18? 3 in 27? 3 in 30?

10. Henry bought 5 three-cent postage stamps: how much did they cost?

11. What will 10 three-cent stamps cost? 7 three-cent stamps? 6 three-cent stamps?

12. How many three-cent postage stamps will cost 15 cents? 30 cents? 24 cents? 27 cents?

13. If a barrel will hold 3 bushels of apples, how many bushels will 7 barrels hold?

14. A drover bought 9 sheep at $3 a head: how many dollars did they cost?

15. What will 8 pencils cost at 3 cents each?

16. There are 3 feet in a yard: how many feet in 6 yards? In 8 yards? 10 yards? 5 yards?

WRITTEN EXERCISES.

Copy and fill:

$3 \times 1 = 3$	$3 \div 3 = 1$	$2 \times 3 =$	$9 \div 3 =$
$3 \times 2 =$	$6 \div 3 =$	$6 \div 3 =$	$12 \div 3 =$
$3 \times 3 =$	$9 \div 3 =$	$4 \times 3 =$	$18 \div 3 =$
$3 \times 4 =$	$12 \div 3 =$	$12 \div 3 =$	$24 \div 3 =$
$3 \times 5 =$	$15 \div 3 =$	$6 \times 3 =$	$30 \div 3 =$
$3 \times 6 =$	$18 \div 3 =$	$18 \div 3 =$	$21 \div 3 =$
$3 \times 7 =$	$21 \div 3 =$	$8 \times 3 =$	$27 \div 3 =$
$3 \times 8 =$	$24 \div 3 =$	$24 \div 3 =$	$15 \div 3 =$
$3 \times 9 =$	$27 \div 3 =$	$10 \times 3 =$	$9 \times 3 =$
$3 \times 10 =$	$30 \div 3 =$	$30 \div 3 =$	$7 \times 3 =$

	1.	2.	3.	4.	5.
	30	30	30	30	30
Multiply	3	4	6	5	7
	90	120			

	6.	**7.**	**8.**	**9.**	**10.**
	31	31	31	31	31
Multiply	4	5	8	9	6

	11.	**12.**	**13.**	**14.**	**15.**	**16.**
	32	32	32	32	32	32
Multiply	4	3	6	7	8	9
			192			

	17.	**18.**	**19.**	**20.**	**21.**	**22.**
	33	33	33	33	33	33
Multiply	4	5	6	8	7	9

LESSON XVIII.

ORAL EXERCISES.

1. How many arrows are twice 4 arrows? 3 times 4 arrows? 4 times 4 arrows?

2. How many arrows are 5 times 4 arrows? 6 times 4 arrows? 7 times 4 arrows?

3. How many wheels are 4 times 4 wheels? 8 times 4 wheels? 9 times 4 wheels? 10 times 4 wheels?

4. How many times 4 wheels in 16 wheels? 4 wheels in 12 wheels? 4 wheels in 20 wheels? 4 wheels in 24 wheels?

5. How many are 5 times 4 dimes? 6 times 4 dimes? 7 times 4 dimes? 10 times 4 dimes?

6. How many are twice 4 cents? 3 times 4 cents? 4 times 4 cents? 5 times 4 cents? 6 times 4 cents?

7. How many are 6 times 4 cents? 7 times 4 cents? 8 times 4 cents? 9 times 4 cents? 10 times 4 cents?

8. How many are 3 times 4? 5 times 4? 6 times 4? *8 times 4?* 9 times 4? 10 times 4?

4	1 time 4?	4 in 4?
4+4, ...	2 times 4?	4 in 8?
4+4+4	3 times 4?	4 in 12?
4+4+4+4	4 times 4?	4 in 16?
4+4+4+4+4	5 times 4?	4 in 20?
4+4+4+4+4+4	6 times 4?	4 in 24?
4+4+4+4+4+4+4. . . .	7 times 4?	4 in 28?
4+4+4+4+4+4+4+4 . .	8 times 4?	4 in 32?
4+4+4+4+4+4+4+4+4 .	9 times 4?	4 in 36?
4+4+4+4+4+4+4+4+4+4	10 times 4?	4 in 40?

9. How many are twice 4? 4 times 4? 6 times 4? 9 times 4? 5 times 4? 7 times 4?

10. How many times 4 in 8? 4 in 16? 4 in 24? 4 in 32? 4 in 40? 4 in 20?

11. How many 4's in 12? 4's in 20? 4's in 28? 4's in 32? 4's in 40? 4's in 36?

12. There are 4 pecks in a bushel: how many pecks in 5 bushels? In 8 bushels? In 10 bushels?

13. There are 4 quarts in a gallon: how many gallons in 16 quarts? In 24 quarts? In 36 quarts?

14. What will 8 loads of wood cost at $4 a load? At $3 a load?

WRITTEN EXERCISES.

Copy and fill:

$4 \times 1 = 4$	$4 \div 4 = 1$	$2 \times 4 =$	$12 \div 4 =$
$4 \times 2 =$	$8 \div 4 =$	$8 \div 4 =$	$20 \div 4 =$
$4 \times 3 =$	$12 \div 4 =$	$4 \times 4 =$	$28 \div 4 =$
$4 \times 4 =$	$16 \div 4 =$	$16 \div 4 =$	$7 \times 4 =$
$4 \times 5 =$	$20 \div 4 =$	$6 \times 4 =$	$9 \times 4 =$
$4 \times 6 =$	$24 \div 4 =$	$24 \div 4 =$	$36 \div 4 =$
$4 \times 7 =$	$28 \div 4 =$	$8 \times 4 =$	$40 \div 4 =$
$4 \times 8 =$	$32 \div 4 =$	$32 \div 4 =$	$36 \div 4 =$
$4 \times 9 =$	$36 \div 4 =$	$10 \times 4 =$	$24 \div 4 =$
$4 \times 10 =$	$40 \div 4 =$	$40 \div 4 =$	$12 \div 4 =$

	1.	2.	3.	4.	5.	6.
Divide	4)20	4)28	4)36	4)12	4)32	4)40
	5					
	7.	**8.**	**9.**	**10.**	**11.**	
	24	34	44	43	42	
Multiply	5	6	4	8	9	
	12.	**13.**	**14.**	**15.**		
Divide	4)88	4)48	4)80	4)44		
	22					
	16.	**17.**	**18.**	**19.**	**20.**	
	203	104	114	244	233	
Multiply	4	8	7	3	4	
	21.	**22.**	**23.**	**24.**	**25.**	
Divide	4)480	4)840	4)808	4)880	4)408	
	120					

LESSON XIX.

ORAL EXERCISES.

1. The leaves of a flower are called *petals*. A violet has 5 petals: how many petals have 2 violets? 3 violets? 4 violets? 5 violets? 6 violets?

2. How many are twice 5? 3 times 5? 4 times 5? 5 times 5? 6 times 5?

3. There are 5 cents in a nickel: how many cents in 5 nickels? In 6 nickels? 7 nickels?

4. How many cents are 5 times 5 cents? 7 times 5 cents? 9 times 5 cents? 8 times 5 cents?

5. How many are 6 times 5? 7 times 5? 8 times 5? 9 times 5? 10 times 5?

5	1 time 5?	5 in 5?
5+5	2 times 5?	5 in 10?
5+5+5	3 times 5?	5 in 15?
5+5+5+5	4 times 5?	5 in 20?
5+5+5+5+5	5 times 5?	5 in 25?
5+5+5+5+5+5	6 times 5?	5 in 30?
5+5+5+5+5+5+5	7 times 5?	5 in 35?
5+5+5+5+5+5+5+5 . .	8 times 5?	5 in 40?
5+5+5+5+5+5+5+5+5 .	9 times 5?	5 in 45?
5+5+5+5+5+5+5+5+5+5	10 times 5?	5 in 50?

6. How many are 3 times 5? 5 times 5? 9 times 5? 7 times 5? 6 times 5? 8 times 5? 10 times 5?

7. How many times 5 in 15? 5 in 25? 5 in 35? 5 in 45? 5 in 50? 5 in 40?

8. How many are 6 times 5? 8 times 5? 2 times 5? 4 times 5? 5 times 5? 10 times 5?

9. How many times 5 in 10? 5 in 20? 5 in 45? 5 in 35? 5 in 25? 5 in 50? 5 in 40?

10. How many 5's in 10? 5's in 20? 5's in 30? 5's in 50? 5's in 40?

11. How many 5's in 15? 5's in 25? 5's in 35? 5's in 45? 5's in 30? 5's in 40? 5's in 50?

12. An orchard has 8 rows of trees, and each row has 5 trees: how many trees in the orchard?

13. A school-room has 7 rows of desks, and 5 desks in each row: how many desks in the room?

14. What will 6 pencils cost at 5 cents apiece?

15. What will 10 oranges cost at 5 cents apiece?

16. What will 9 hats cost at $5 apiece?

17. At 5 cents apiece, how many oranges can you buy for 25 cents? For 50 cents? 40 cents?

18. How many pencils, at 5 cents each, can be bought for 35 cents? For 40 cents? 50 cents?

19. There are 5 trees in a row: how many trees in 3 rows? 5 rows? 7 rows? 9 rows? 10 rows?

WRITTEN EXERCISES.

Copy and fill:

$5 \times 1 = 5$	$5 \div 5 = 1$	$6 \times 5 =$	$5 \times 9 =$
$5 \times 2 =$	$10 \div 5 =$	$30 \div 5 =$	$40 \div 5$
$5 \times 3 =$	$15 \div 5 =$	$7 \times 5 =$	$36 \div 4$
$5 \times 4 =$	$20 \div 5 =$	$35 \div 5 =$	4×8
$5 \times 5 =$	$25 \div 5 =$	$10 \times 5 =$	$30 \div 5$
$5 \times 6 =$	$30 \div 5 =$	$50 \div 5 =$	$27 \div 3$
$5 \times 7 =$	$35 \div 5 =$	$5 \times 5 =$	3×8
$5 \times 8 =$	$40 \div 5 =$	$25 \div 5 =$	$50 \div 5$
$5 \times 9 =$	$45 \div 5 =$	$9 \times 5 =$	$45 \div 5$
$5 \times 10 =$	$50 \div 5 =$	$45 \div 5 =$	$28 \div 4$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	25	35	45	54	53	52
Multiply	4	7	8	6	7	9

	7.	**8.**	**9.**	**10.**	**11.**
	125	145	254	205	115
Multiply	6	5	3	4	8

	12.	**13.**	**14.**	**15.**	**16.**
Divide	5)255	5)305	5)405	5)455	5)525

	17.	**18.**	**19.**	**20.**	**21.**
Divide	5)125	5)265	5)605	5)705	5)315

LESSON XX.

ORAL EXERCISES.

1. A fly has 6 legs: how many legs have 2 flies? 3 flies?

2. How many legs have 4 flies? 5 flies? *6 flies?* 8 flies? 10 flies?

3. How many are twice 6? 3 times 6? 4 times 6? 5 times 6? 6 times 6? 8 times 6? 10 times 6?

4. How many spools are 7 times 6 spools? 8 times 6 spools? 9 times 6 spools? 10 times 6 spools?

5. How many are 6 times 6? 7 times 6? 8 times 6? 9 times 6? 10 times 6?

6	1 time 6?	6 in 6?
6+6	2 times 6?	6 in 12?
6+6+6	3 times 6?	6 in 18?
6+6+6+6	4 times 6?	6 in 24?
6+6+6+6+6	5 times 6?	6 in 30?
6+6+6+6+6+6	6 times 6?	6 in 36?
6+6+6+6+6+6+6	7 times 6?	6 in 42?
6+6+6+6+6+6+6+6 . .	8 times 6?	6 in 48?
6+6+6+6+6+6+6+6+6 .	9 times 6?	6 in 54?
6+6+6+6+6+6+6+6+6+6	10 times 6?	6 in 60?

6. How many are twice 6? 4 times 6? 6 times 6? 8 times 6? 10 times 6? 5 times 6? 7 times 6?

7. How many are 3 times 6? 5 times 6? 7 times 6? 9 times 6? 8 times 6? 10 times 6? 6 times 6?

8. How many times 6 in 24? 6 in 36? 6 in 48? 6 in 60? 6 in 30? 6 in 42? 6 in 18?

9. How many times 6 in 12? 6 in 18? 6 in 24? 6 in 30? 6 in 36? 6 in 42? 6 in 48? 6 in 54?

10. How many 6's in 18? 6's in 30? 6's in 42? 6's in 60? 6's in 54? 6's in 36? 6's in 48?

11. Mary wrote 5 columns of figures, and each column had 6 figures: how many figures did she write?

12. Harry wrote 8 words, and each word contained 6 letters: how many letters did he write?

13. What will 6 oranges cost, at 6 cents apiece?

14. A school-house has 9 windows, and each window has 6 panes of glass: how many panes of glass in the house?

15. A gardener has 60 stalks of celery: how many bunches can he make if he put 6 stalks in each bunch?

16. What will 8 bunches of radishes cost, at 6 cents a bunch? At 5 cents a bunch?

WRITTEN EXERCISES.

Copy and fill:

$6 \times 1 = 6$	$6 \div 6 = 1$	$6 \times 7 =$	$6 \times 7 = 42$
$6 \times 2 =$	$12 \div 6 =$	$42 \div 6 =$	$5 \times \quad = 40$
$6 \times 3 =$	$18 \div 6 =$	$6 \times 9 =$	$6 \times \quad = 54$
$6 \times 4 =$	$24 \div 6 =$	$54 \div 6 =$	$6 \times \quad = 36$
$6 \times 5 =$	$30 \div 6 =$	$6 \times 8 =$	$5 \times \quad = 50$
$6 \times 6 =$	$36 \div 6 =$	$48 \div 6 =$	$6 \times \quad = 48$
$6 \times 7 =$	$42 \div 6 =$	$10 \times 6 =$	$4 \times \quad = 36$
$6 \times 8 =$	$48 \div 6 =$	$60 \div 6 =$	$5 \times \quad = 35$
$6 \times 9 =$	$54 \div 6 =$	$5 \times 6 =$	$5 \times \quad = 45$
$6 \times 10 =$	$60 \div 6 =$	$30 \div 6 =$	$6 \times \quad = 60$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	26	36	46	56	66	63
Multiply	7	8	9	8	6	7

	7.	**8.**	**9.**	**10.**	**11.**
	136	246	106	226	135
Multiply	5	4	9	3	6

	12.	**13.**	**14.**	**15.**	**16.**
Divide	6)426	6)546	6)660	6)306	6)480

	17.	**18.**	**19.**	**20.**	**21.**
Divide	6)372	6)268	6)324	6)444	6)504

	22.	**23.**	**24.**	**25.**	**26.**
Divide	6)498	6)384	6)564	6)732	6)852

LESSON XXI.

ORAL EXERCISES.

1. This rose leaf has 7 blades: how many blades have two leaves?

2. How many blades are 3 times 7 blades? 4 times 7 blades? 5 times 7 blades? 6 times 7 blades?

3. How many are twice 7 leaves? 3 times 7 leaves? 4 times 7 leaves? 5 times 7 leaves? 6 times 7 leaves?

4. How many grapes are 7 times 7 grapes? 8 times 7 grapes? 9 times 7 grapes? 10 times 7 grapes? 4 times 7 grapes?

7	1 time 7? 7 in 7?
7+7.	2 times 7? 7 in 14?
7+7+7	3 times 7? 7 in 21?
7+7+7+7	4 times 7? 7 in 28?
7+7+7+7+7	5 times 7? 7 in 35?
7+7+7+7+7+7	6 times 7? 7 in 42?
7+7+7+7+7+7+7. . . .	7 times 7? 7 in 49?
7+7+7+7+7+7+7+7 . .	8 times 7? 7 in 56?
7+7+7+7+7+7+7+7+7 .	9 times 7? 7 in 63?
7+7+7+7+7+7+7+7+7+7	10 times 7? 7 in 70?

5. How many are 6 times 7? 7 times 7? 8 times 7? 9 times 7? 10 times 7? 4 times 7?

6. How many are twice 7? 4 times 7? 6 times 7? 8 times 7? 10 times 7? 7 times 7? 5 times 7?

7. How many are 3 times 7? 5 times 7? 7 times 7? 9 times 7? 6 times 7? 8 times 7?

8. How many times 7 in 14? 7 in 21? 7 in 35? 7 in 42? 7 in 49? 7 in 56? 7 in 63?

9. How many 7's in 14? 7's in 21? 7's in 35? 7's in 28? 7's in 56? 7's in 63? 7's in 49? 7's in 70?

10. There are 7 days in a week: how many days in 7 weeks? How many days in 9 weeks?

11. There are 7 days in a week: how many weeks in 28 days? In 56 days? 49 days?

12. What will 8 quarts of milk cost at 7 cents a quart?

13. At 7 cents a quart, how many quarts of milk can be bought for 63 cents? For 56 cents?

WRITTEN EXERCISES.

Copy and fill:

$7 \times 1 =$	$7 \div 7 =$	$4 \times 7 =$	$35 \div \quad = 7$
$7 \times 2 =$	$14 \div 7 =$	$28 \div 7 =$	$42 \div \quad = 7$
$7 \times 3 =$	$21 \div 7 =$	$6 \times 7 =$	$35 \div \quad = 7$
$7 \times 4 =$	$28 \div 7 =$	$42 \div 7 =$	$28 \div \quad = 7$
$7 \times 5 =$	$35 \div 7 =$	$8 \times 7 =$	$49 \div \quad = 7$
$7 \times 6 =$	$42 \div 7 =$	$56 \div 7 =$	$56 \div \quad = 8$
$7 \times 7 =$	$49 \div 7 =$	$9 \times 7 =$	$63 \div \quad = 9$
$7 \times 8 =$	$56 \div 7 =$	$63 \div 7 =$	$28 \div \quad = 4$
$7 \times 9 =$	$63 \div 7 =$	$7 \times 7 =$	$42 \div \quad = 6$
$7 \times 10 =$	$70 \div 7 =$	$49 \div 7 =$	$56 \div \quad = 7$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
	37	47	73	74	75	76	77
Multiply	8	9	6	7	8	7	9

	8.	**9.**	**10.**	**11.**	**12.**
	137	176	207	127	107
Multiply	6	5	4	7	8

	13.	**14.**	**15.**	**16.**	**17.**
Divide	8)120	8)168	8)208	8)272	8)264

	18.	**19.**	**20.**	**21.**	**22.**
Divide	6)252	7)364	8)280	5)375	4)380

LESSON XXII.

ORAL EXERCISES.

1. There are 8 panes of glass in a window: how many panes in 2 windows? In 3 windows?

2. How many are 3 times 8 panes? 4 times 8 panes? 5 times 8 panes? 6 times 8 panes?

3. How many are 2 times 8 pins? 3 times 8 pins? 4 times 8 pins? 5 times 8 pins?

4. How many pins are 6 times 8 pins? 7 times 8 pins? 9 times 8 pins? 10 times 8 pins?

8	1 time 8? 8 in 8?
8+8.	2 times 8? 8 in 16?
8+8+8	3 times 8? 8 in 24?
8+8+8+8	4 times 8? 8 in 32?
8+8+8+8+8	5 times 8? 8 in 40?
8+8+8+8+8+8	6 times 8? 8 in 48?
8+8+8+8+8+8+8	7 times 8? 8 in 56?
8+8+8+8+8+8+8+8 . .	8 times 8? 8 in 64?
8+8+8+8+8+8+8+8+8 .	9 times 8? 8 in 72?
8+8+8+8+8+8+8+8+8+8	10 times 8? 8 in 80?

5. How many are 5 times 8? 6 times 8? 7 times 8? 9 times 8? 10 times 8?

6. How many are 3 times 8? 5 times 8? 7 times 8? 9 times 8? 6 times 8? 4 times 8?

7. How many are 4 times 8? 6 times 8? 8 times 8? 10 times 8? 7 times 8? 9 times 8?

8. How many times 8 in 24? 8 in 40? 8 in 56? 8 in 72? 8 in 80?

9. How many times 8 in 32? 8 in 48? 8 in 64? 8 in 80? 8 in 56? 8 in 40?

10. How many 8's in 16? 8's in 40? 8's in 32? 8's in 56? 8's in 24? 8's in 64? 8's in 80? 8's in 72?

11. There are 8 quarts in a peck: how many quarts in 3 pecks? 5 pecks? 8 pecks? 6 pecks? 9 pecks?

12. If 8 quarts make a peck, how many pecks in 16 quarts? In 24 quarts? 40 quarts?

13. At 8 cents apiece, how much will 4 copy-books cost? 7 copy-books? 10 copy-books?

14. At 8 cents a yard, how many yards of muslin can be bought for 40 cents? For 80 cents?

WRITTEN EXERCISES.

Copy and fill:

$8 \times 1 =$	$8 \div 8 =$	$6 \times 8 =$	$7 \times \quad = 56$
$8 \times 2 =$	$16 \div 8 =$	$48 \div 8 =$	$8 \times \quad = 72$
$8 \times 3 =$	$24 \div 8 =$	$8 \times 8 =$	$7 \times \quad = 42$
$8 \times 4 =$	$32 \div 8 =$	$64 \div 8 =$	$6 \times \quad = 54$
$8 \times 5 =$	$40 \div 8 =$	$5 \times 8 =$	$7 \times \quad = 63$
$8 \times 6 =$	$48 \div 8 =$	$40 \div 8 =$	$8 \times \quad = 56$
$8 \times 7 =$	$56 \div 8 =$	$9 \times 8 =$	$8 \times \quad = 80$
$8 \times 8 =$	$64 \div 8 =$	$72 \div 8 =$	$5 \times \quad = 45$
$6 \times 9 =$	$72 \div 8 =$	$7 \times 8 =$	$6 \times \quad = 48$
$8 \times 10 =$	$80 \div 8 =$	$56 \div 8 =$	$10 \times \quad = 70$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	28	48	88	68	78	58
Multiply	8	7	6	9	7	8

	7.	**8.**	**9.**	**10.**	**11.**
	\$138	\$217	\$178	\$108	\$117
Multiply	6	4	5	9	8

	12.	**13.**	**14.**	**15.**	**16.**
Divide	8)168	8)248	8)208	8)272	8)264

	17.	**18.**	**19.**	**20.**	**21.**
Divide	*8)\$456*	8)\$664	8)\$768	8)\$448	8)\$504

LESSON XXIII.

ORAL EXERCISES.

1. There are 9 ears of corn in a bunch: how many ears in 2 bunches? 3 bunches?

2. How many ears are twice 9 ears? 3 times 9 ears? 4 times 9 ears? 5 times 9 ears?

3. How many are 2 times 9? 3 times 9? 4 times 9? 5 times 9?

4. How many are 6 times 9 trees? 7 times 9 trees? 8 times 9 trees? 9 times 9 trees? 10 times 9 trees? 5 times 9 trees?

9	1 time 9?	9 in 9?
9+9	2 times 9?	9 in 18?
9+9+9	3 times 9?	9 in 27?
9+9+9+9	4 times 9?	9 in 36?
9+9+9+9+9	5 times 9?	9 in 45?
9+9+9+9+9+9	6 times 9?	9 in 54?
9+9+9+9+9+9+9	7 times 9?	9 in 63?
9+9+9+9+9+9+9+9 . .	8 times 9?	9 in 72?
9+9+9+9+9+9+9+9+9 .	9 times 9?	9 in 81?
9+9+9+9+9+9+9+9+9+9	10 times 9?	9 in 90?

5. How many are 5 times 9? 6 times 9? 7 times 9? 8 times 9? 9 times 9? 10 times 9?

6. How many are 3 times 9? 5 times 9? 7 times 9? 4 times 9? 6 times 9? 8 times 9? 10 times 9?

7. How many are 4 times 9? 6 times 9? 8 times 9? 10 times 9? 9 times 7? 9 times 6? 9 times 8?

8. How many times 9 in 36? 9 in 45? 9 in 27? 9 in 54? 9 in 72? 9 in 18?

9. How many times 9 in 63? 9 in 72? 9 in 90? 9 in 36? 9 in 54? 9 in 81?

10. How many 9's in 18? 9's in 36? 9's in 54? 9's in 72? 9's in 90? 9's in 27? 9's in 45? 9's in 81?

11. There are 7 rows of trees in a peach orchard, and 9 trees in each row: how many trees in the orchard?

12. A farmer filled 8 bags with wheat, putting 9 pecks in each bag: how many pecks of wheat in the bags?

13. A drover bought 10 hogs at $9 apiece: how much did he pay for them?

14. What will 6 heads of cabbage cost at 9 cents a head? What will 10 heads cost?

WRITTEN EXERCISES.

Copy and fill:

$9 \times 1 =$	$9 \div 9 =$	$7 \times 9 =$	$6 \times \quad = 42$
$9 \times 2 =$	$18 \div 9 =$	$63 \div 9 =$	$6 \times \quad = 54$
$9 \times 3 =$	$27 \div 9 =$	$9 \times 9 =$	$7 \times \quad = 56$
$9 \times 4 =$	$36 \div 9 =$	$81 \div 9 =$	$7 \times \quad = 63$
$9 \times 5 =$	$45 \div 9 =$	$5 \times 9 =$	$8 \times \quad = 72$
$9 \times 6 =$	$64 \div 9 =$	$45 \div 9 =$	$8 \times \quad = 80$
$9 \times 7 =$	$73 \div 9 =$	$8 \times 9 =$	$9 \times \quad = 54$
$9 \times 8 =$	$82 \div 9 =$	$72 \div 9 =$	$9 \times \quad = 72$
$9 \times 9 =$	$91 \div 9 =$	$10 \times 9 =$	$9 \times \quad = 63$
$9 \times 10 =$	$50 \div 9 =$	$90 \div 9 =$	$9 \times \quad = 81$

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
	$49	$69	$89	$39	$59	$79
Multiply	5	4	3	7	9	6

	7.	**8.**	**9.**	**10.**	**11.**
	149	169	189	139	109
Multiply	5	4	3	7	9

	12.	**13.**	**14.**	**15.**	**16.**
Divide	9)$189	9)$288	9)$396	9)$567	9)$583

	17.	**18.**	**19.**	**20.**	**21.**
Divide	*9)747*	9)846	9)774	9)693	9)684

LESSON XXIV.

ORAL EXERCISES.

1. There are 10 cents in one dime: how many cents in 2 dimes? In 3 dimes? 4 dimes? 5 dimes? 6 dimes?

2. How many are 6 times 10 dimes? 7 times 10 dimes? 8 times 10 dimes? 9 times 10 dimes? 10 times 10 dimes?

1 time 10?	10 in 10?
2 times 10?	10 in 20?
3 times 10?	10 in 30?
4 times 10?	10 in 40?
5 times 10?	10 in 50?
6 times 10?	10 in 60?
7 times 10?	10 in 70?
8 times 10?	10 in 80?
9 times 10?	10 in 90?
10 times 10?	10 in 100?

3. How many are 5 times 10? 7 times 10? 9 times 10? 8 times 10? 6 times 10? 4 times 10?

4. How many times 10 in 20? 10 in 30? 10 in 50? 10 in 40? 10 in 60? 10 in 70? 10 in 90? 10 in 80?

5. How many 10's in 70? 10's in 80? 10's in 100? 10's in 60? 10's in 50? 10's in 40? 10's in 90?

6. There are 10 cents in 1 dime: how many cents in 6 dimes? In 8 dimes? 7 dimes? 10 dimes?

7. There are 10 dimes in 1 dollar: how many dimes in 4 dollars? In 7 dollars? 5 dollars? 8 dollars?

8. How many dimes in 40 cents? In 60 cents? 80 cents? 100 cents? 50 cents? 70 cents?

9. How many cents in 5 dimes? 8 dimes? 3 dimes? 4 dimes? 10 dimes? 2 dimes? 9 dimes?

10. Here is a woman selling postage stamps. Each stamp is worth 3 cents, and there are 10 stamps in each row: how much is 1 row worth? 2 rows? 5 rows?

11. What is a sheet containing 10 rows of 3-cent stamps worth? 2 sheets? 3 sheets?

WRITTEN EXERCISES.

Copy and fill;

$10 \times 1 =$	$10 \div 10 =$	$7 \times 9 =$	$63 \div \quad = 7$
$10 \times 2 =$	$20 \div 10 =$	$9 \times 8 =$	$72 \div \quad = 9$
$10 \times 3 =$	$30 \div 10 =$	$7 \times 8 =$	$81 \div \quad = 9$
$10 \times 4 =$	$40 \div 10 =$	$8 \times 6 =$	$90 \div \quad = 9$
$10 \times 5 =$	$50 \div 10 =$	$10 \times 7 =$	$64 \div \quad = 8$
$10 \times 6 =$	$60 \div 10 =$	$10 \times 9 =$	$80 \div \quad = 8$
$10 \times 7 =$	$70 \div 10 =$	$6 \times 9 =$	$63 \div \quad = 7$
$10 \times 8 =$	$80 \div 10 =$	$9 \times 7 =$	$56 \div \quad = 7$
$10 \times 9 =$	$90 \div 10 =$	$6 \times 8 =$	$70 \div \quad = 7$
$10 \times 10 =$	$100 \div 10 =$	$8 \times 9 =$	$49 \div \quad = 7$

	1.	**2.**	**3.**	**4.**	**5.**
Divide	5)450	5)665	5)725	6)174	6)906
	6.	**7.**	**8.**	**9.**	**10.**
Divide	7)231	7)266	7)476	7)189	7)329
	11.	**12.**	**13.**	**14.**	**15.**
Divide	8)272	8)408	8)512	8)328	8)504
	16.	**17.**	**18.**	**19.**	**20.**
Divide	9)279	9)396	9)477	9)648	9)864

LESSON XXV.

EQUAL PARTS OF NUMBERS.

1. If a melon be cut into two *equal* pieces, what part of the melon will each piece be?

2. If a group of melons be divided into two equal parts, what part of the group will each of these parts be?

3. One half of 6 melons are how many melons? One half of twelve melons? One half of 16 melons? One half of 20 melons?

4. If an apple be divided into four equal parts, what is each part called?

5. What is one of the *five* equal parts of an object called? One of the *six* equal parts? One of the *ten* equal parts?

6. What is one fourth of 8 cents? One fourth of 16 cents? One fourth of 28 cents? Of 40 cents? Of 32 cents? Of 12 cents?

7. What is one fourth of 20? One fourth of 28? One fourth of 36? One fourth of 40?

8. How much is one fifth of 15? Of 25? 35? 20? 30? 45? 40? 50?

9. How much is one seventh of 21? Of 28? 35? 56? 70? 63?

10. How much is one ninth of 27? Of 45? 63? 54? 36? 72? 90? 81?

11. How much is one sixth of $24? Of $48? $36? $60? 54? 42? 30?

12. How much is one eighth of 16? Of 32? 56? 48? 72? 80? 64?

13. How much is one tenth of 20? Of 40? 60? 80? 50? 70? 90? 100?

LESSON XXVI.

REVIEW.

ORAL EXERCISES.

Read and complete:

1.	$12 = 4 \times$	$12 = 3 \times$	$12 \div 4 =$	$12 \div 3 =$
2.	$14 = 7 \times$	$14 = 2 \times$	$14 \div 7 =$	$14 \div 2 =$
3.	$24 = 6 \times$	$24 = 4 \times$	$24 \div 6 =$	$24 \div 4 =$
4.	$28 = 7 \times$	$28 = 4 \times$	$28 \div 7 =$	$28 \div 4 =$
5.	$21 = 7 \times$	$21 = 3 \times$	$21 \div 7 =$	$21 \div 3 =$
6.	$20 = 5 \times$	$20 = 4 \times$	$20 \div 5 =$	$20 \div 4 =$
7.	$35 = 7 \times$	$35 = 5 \times$	$35 \div 7 =$	$35 \div 5 =$
8.	$30 = 6 \times$	$30 = 5 \times$	$30 \div 6 =$	$30 \div 5 =$
9.	$18 = 6 \times$	$18 = 3 \times$	$18 \div 6 =$	$18 \div 3 =$
10.	$36 = 9 \times$	$36 = 4 \times$	$36 \div 9 =$	$36 \div 4 =$
11.	$54 = 9 \times$	$54 = 6 \times$	$54 \div 9 =$	$54 \div 6 =$
12.	$48 = 8 \times$	$48 = 6 \times$	$48 \div 8 =$	$48 \div 6 =$
13.	$45 = 9 \times$	$45 = 5 \times$	$45 \div 9 =$	$45 \div 5 =$
14.	$56 = 8 \times$	$56 = 7 \times$	$56 \div 8 =$	$56 \div 7 =$
15.	$27 = 9 \times$	$27 = 3 \times$	$27 \div 9 =$	$27 \div 3 =$
16.	$42 = 7 \times$	$42 = 6 \times$	$42 \div 7 =$	$42 \div 6 =$
17.	$32 = 8 \times$	$32 = 4 \times$	$32 \div 8 =$	$32 \div 4 =$
18.	$40 = 8 \times$	$40 = 5 \times$	$40 \div 8 =$	$40 \div 5 =$
19.	$72 = 9 \times$	$72 = 8 \times$	$72 \div 9 =$	$72 \div 8 =$
20.	$63 = 9 \times$	$63 = 7 \times$	$63 \div 9 =$	$63 \div 7 =$
21.	$16 = 8 \times$	$16 = 2 \times$	$16 \div 8 =$	$16 \div 2 =$
22.	$40 = 10 \times$	$40 = 4 \times$	$40 \div 10 =$	$40 \div 4 =$
23.	$60 = 10 \times$	$60 = 6 \times$	$60 \div 10 =$	$60 \div 6 =$
24.	$70 = 10 \times$	$70 = 7 \times$	$70 \div 10 =$	$70 \div 7 =$
25.	$90 = 10 \times$	$90 = 9 \times$	$90 \div 10 =$	$90 \div 9 =$
26.	$80 = 10 \times$	$80 = 8 \times$	$80 \div 10 =$	$80 \div 8 =$
27.	$50 = 10 \times$	$50 = 5 \times$	$50 \div 10 =$	$50 \div 5 =$
28.	$16 = 4 \times$	$16 \div 4 =$	$25 = 5 \times$	$25 \div 5 =$
29.	$36 = 6 \times$	$36 \div 6 =$	$49 = 7 \times$	$49 \div 7 =$
30.	$64 = 8 \times$	$64 \div 8 =$	$81 = 9 \times$	$81 \div 9 =$

BLACKBOARD EXERCISES.

1.	2.
$5 + 4, - 4, \times 4, \div 4 =$	$6 + 7, - 7, \times 7, \div 7 =$
$5 + 7, - 7, \times 7, \div 7 =$	$6 + 9, - 9, \times 9, \div 9 =$
$5 + 8, - 8, \times 8, \div 8 =$	$7 + 7, - 7, \times 7, \div 7 =$
$5 + 9, - 9, \times 9, \div 9 =$	$7 + 6, - 6, \times 6, \div 6 =$
$6 + 5, - 5, \times 5, \div 5 =$	$8 + 8, - 8, \times 8, \div 8 =$
$6 + 8, - 8, \times 8, \div 8 =$	$7 + 5, - 5, \times 5, \div 5 =$
$7 + 9, - 9, \times 9, \div 9 =$	$8 + 6, - 6, \times 6, \div 6 =$
$7 + 8, - 8, \times 8, \div 8 =$	$9 + 5, - 5, \times 5, \div 5 =$
$8 + 7, - 7, \times 7, \div 7 =$	$9 + 7, - 7, \times 7, \div 7 =$
$8 + 9, - 9, \times 9, \div 9 =$	$9 + 9, - 9, \times 9, \div 9 =$
$9 + 8, - 8, \times 8, \div 8 =$	$9 + 6, - 6, \times 6, \div 6 =$

3. $5 + 7, - 4, \times 7, + 4, \div 6, \div 2, \times 9, + 5, + 6, \div 8 =$ what?

4. $9 \times 6, + 7, - 5, \div 8, + 5, \times 6, + 9, \div 9, \times 5, + 4, \div 7 =$ what?

5. $62 - 6, \div 8, \times 6, + 9, - 6, \div 9, + 4, \times 8, + 9, \div 9, \div 3 =$ what?

6. $72 \div 9, \times 9, + 9, \div 9, \div 3, \times 9, + 3, \div 5, \div 3, \times 6, + 8, \div 4 =$ what?

7. $57 - 3, \div 9, \times 8, + 9, + 7, \div 8, \times 10, + 1, \div 9, \div 3 =$ what?

8. $56 \div 7, \times 5, + 8, - 3, \div 9, \times 4, \times 3, + 3, \div 7, \times 8 =$ what?

9. $63 \div 7, \times 6, + 6, \div 10, \times 8, + 8, \div 8, \times 9, + 7, \div 10 =$ what?

10. $75 + 6, \div 9, \times 8, + 8, \div 8, \times 6, + 3, \div 7, \times 6, - 8 =$ what?

11. $81 \div 9, \times 7, + 7, \div 10, \times 8, + 4, \div 6, \times 9 =$ what?

NOTE.—The comma (,) is used in the above exercises to indicate that the operations expressed by the signs +, —, ×, and ÷, are to be performed *in the order in which they occur from left to right.* If only + and — were used, the comma would not be needed.

Multiplication Table.

1 × 1 = 1	1 × 2 = 2	1 × 3 = 3	1 × 4 = 4
2 × 1 = 2	2 × 2 = 4	2 × 3 = 6	2 × 4 = 8
3 × 1 = 3	3 × 2 = 6	3 × 3 = 9	3 × 4 = 12
4 × 1 = 4	4 × 2 = 8	4 × 3 = 12	4 × 4 = 16
5 × 1 = 5	5 × 2 = 10	5 × 3 = 15	5 × 4 = 20
6 × 1 = 6	6 × 2 = 12	6 × 3 = 18	6 × 4 = 24
7 × 1 = 7	7 × 2 = 14	7 × 3 = 21	7 × 4 = 28
8 × 1 = 8	8 × 2 = 16	8 × 3 = 24	8 × 4 = 32
9 × 1 = 9	9 × 2 = 18	9 × 3 = 27	9 × 4 = 36
10 × 1 = 10	10 × 2 = 20	10 × 3 = 30	10 × 4 = 40
11 × 1 = 11	11 × 2 = 22	11 × 3 = 33	11 × 4 = 44
12 × 1 = 12	12 × 2 = 24	12 × 3 = 36	12 × 4 = 48
1 × 5 = 5	1 × 6 = 6	1 × 7 = 7	1 × 8 = 8
2 × 5 = 10	2 × 6 = 12	2 × 7 = 14	2 × 8 = 16
3 × 5 = 15	3 × 6 = 18	3 × 7 = 21	3 × 8 = 24
4 × 5 = 20	4 × 6 = 24	4 × 7 = 28	4 × 8 = 32
5 × 5 = 25	5 × 6 = 30	5 × 7 = 35	5 × 8 = 40
6 × 5 = 30	6 × 6 = 36	6 × 7 = 42	6 × 8 = 48
7 × 5 = 35	7 × 6 = 42	7 × 7 = 49	7 × 8 = 56
8 × 5 = 40	8 × 6 = 48	8 × 7 = 56	8 × 8 = 64
9 × 5 = 45	9 × 6 = 54	9 × 7 = 63	9 × 8 = 72
10 × 5 = 50	10 × 6 = 60	10 × 7 = 70	10 × 8 = 80
11 × 5 = 55	11 × 6 = 66	11 × 7 = 77	11 × 8 = 88
12 × 5 = 60	12 × 6 = 72	12 × 7 = 84	12 × 8 = 96
1 × 9 = 9	1 × 10 = 10	1 × 11 = 11	1 × 12 = 12
2 × 9 = 18	2 × 10 = 20	2 × 11 = 22	2 × 12 = 24
3 × 9 = 27	3 × 10 = 30	3 × 11 = 33	3 × 12 = 36
4 × 9 = 36	4 × 10 = 40	4 × 11 = 44	4 × 12 = 48
5 × 9 = 45	5 × 10 = 50	5 × 11 = 55	5 × 12 = 60
6 × 9 = 54	6 × 10 = 60	6 × 11 = 66	6 × 12 = 72
7 × 9 = 63	7 × 10 = 70	7 × 11 = 77	7 × 12 = 84
8 × 9 = 72	8 × 10 = 80	8 × 11 = 88	8 × 12 = 96
9 × 9 = 81	9 × 10 = 90	9 × 11 = 99	9 × 12 = 108
10 × 9 = 90	10 × 10 = 100	10 × 11 = 110	10 × 12 = 120
11 × 9 = 99	11 × 10 = 110	11 × 11 = 121	11 × 12 = 132
12 × 9 = 108	12 × 10 = 120	12 × 11 = 132	12 × 12 = 144

Note.—The sign ×, in the above table, is to be read *time* when it follows 1, and *times* when it follows other numbers.

PART II.

NUMERATION AND NOTATION.

LESSON I.

UNITS' PERIOD.

ARTICLE 1. When a number is expressed by two figures, the first or right-hand figure denotes *units*, and the second figure denotes *tens*. In 25, the 2 denotes tens, and the 5, units.

1. How many tens and how many units in 37? In 57? 64? 88? 94? 78? 80? 90?

ART. 2. In reading a number expressed by two figures, the tens and units are read together as so many units. Thus, 45 is read *forty-five units*, or, more briefly, *forty-five.*

2. Read 29; 48; 76; 60; 84; 90; 67; 58; 75.

ART. 3. When a number is expressed by three figures, the third or left-hand figure denotes *hundreds.* In 245, the 2 denotes hundreds; the 4, tens; and the 5, units.

3. How many hundreds, how many tens, and how many units in 426? 738? 406? 560? 700? 756?

ART. 4. In reading a number expressed by three figures, the hundreds, tens, and units are read together as so many units. Thus, 245 is read *two hundred and forty-five*, or, more briefly, *two hundred forty-five.*

4. Read 424; 740; 706; 804; 500; 660; 999.

WRITTEN EXERCISES.

1. Write in words: 300; 350; 465; 506; 708; 750.

Express in figures the following numbers:

2.	3.
Two hundred.	Four hundred and five.
Five hundred.	Five hundred and six.
Seven hundred.	Six hundred and four.
Three hundred forty.	Four hundred forty-five.
Six hundred seventy.	Eight hundred thirty-seven.
Nine hundred thirty.	Nine hundred twenty-seven.

4. Express in figures the numbers composed of three hundreds, five tens, and four units; six hundreds, four tens, and three units; five hundreds, seven tens, and no units.

5. Express in figures the numbers composed of eight hundreds and six tens; five hundreds and four tens; seven hundreds and five units; two hundreds and six units.

6. What number is composed of 3 hundreds, 0 tens, and 6 units? 2 hundreds and 3 tens? 4 hundreds and 6 units? 5 hundreds and 8 tens?

7. What number is composed of 5 tens and 8 units? 6 hundreds and 5 units? 7 hundreds and 6 tens?

LESSON II.

TWO PERIODS—UNITS AND THOUSANDS.

ORAL EXERCISES.

ART. 5. When a number is expressed by four figures, the fourth or left-hand figure denotes *thousands.* In 4,235, the 4 denotes thousands.

1. How many thousands in 4,000? 6,000? 8,000? 5,400? 7,540? 9,675? 6,084? 7,830?

Read the following numbers:

2.	3.	4.	5.	6.
1,000	2,200	1,020	2,007	3,432
3,000	4,400	3,040	4,001	4,568
5,000	6,600	5,060	5,003	5,608
7,000	8,800	7,090	6,005	7,893
9,000	9,900	9,070	8,009	9,890

ART. 6. When a number is expressed by five figures, the fifth or left-hand figure denotes tens of thousands, or *ten-thousands.* In 45,670, the 4 denotes ten-thousands, and the 5, thousands.

7. How many ten-thousands and how many thousands in 48,500? 85,350? 50,480? 60,070? 92,300?

In reading a number expressed by five figures, the fifth and fourth figures are read together as so many thousands. Thus, 45,400 is read *forty-five thousand four hundred.*

Read the following numbers:

8.	9.	10.	11.
10,000	21,000	34,400	53,333
30,000	44,000	53,440	16,089
50,000	63,000	67,444	99,008
70,000	84,000	48,307	28,045
90,000	99,000	39,600	67,909

ART. 7. When a number is expressed by six figures, the sixth or left-hand figure denotes hundreds of thousands, or *hundred-thousands.* In 534,000, the 5 denotes hundred-thousands.

12. How many hundred-thousands, how many ten-thousands, and how many thousands in 328,000? 650,000? 508,080? 607,800? 350,307? 470,386?

In reading a number expressed by six figures, the

sixth, fifth, and fourth figures are read together as thousands. Thus, 452,500 is read *four hundred fifty-two thousand five hundred.*

Read the following numbers;

13.	14.	15.	16.
200,000	250,000	845,630	603,408
400,000	360,000	803,084	490,732
600,000	580,000	760,432	308,400
800,000	730,000	900,425	600,550
900,000	960,000	807,708	707,700

ART. 8. The first group of three figures; viz, units, tens, and hundreds, constitutes the first or *Units' Period.*

The second group of three figures; viz, thousands, ten-thousands, and hundred-thousands, constitutes the second or *Thousands' Period.*

The two periods may be separated by a comma, as in the examples above.

17. Read the thousands' period in the 13th, 14th, 15th, and 16th examples above.

18. Read the units' period in the same examples.

WRITTEN EXERCISES.

19. Write in words: 3000; 4060; 7086; 6066; 8006.

20. Write in words: 20000; 25000; 40500; 36300; 48080; 30400.

21. Write in words: 300000; 440000; 506000; 308400; 450340.

Express the following in figures as one number:

22. 87 thousand 327 units.

23. 70 thousand 75 units.

24. 200 thousand 2 units.

25. *8 thousand* 80 units.

26. 250 thousand 50 units.
27. 305 thousand 305 units.
28. 708 thousand 8 units.
29. 560 thousand 360 units.
30. 780 thousand 600 units.
31. 80 thousand 80 units.

Express the following in figures;

32. Three thousand four hundred.
33. Seven thousand six hundred fifty.
34. Two thousand four hundred forty-two.
35. Four thousand five hundred and six.
36. Six thousand five hundred twenty-five.
37. Nine thousand.
38. Nine hundred ninety-two.
39. Forty-five thousand five hundred.
40. Sixty thousand seven hundred ninety.
41. Thirty-eight thousand twenty-two.
42. Ninety-six thousand four hundred fifteen.
43. Sixteen thousand ninety-eight.
44. Forty thousand.
45. Fifty-four thousand.
46. Eight thousand eighty.
47. Twenty thousand and two.
48. Sixty-two thousand six hundred.
49. Seventy-three thousand thirty-four.
50. Ninety thousand seventy.
51. Nine hundred sixty-two thousand.
52. Six hundred thousand five hundred.
53. Four hundred fifty thousand two hundred.
54. Eight hundred forty-two thousand twenty-seven.
55. Seven hundred sixty thousand seven hundred six.
56. Nine hundred thousand.
57. Nine hundred forty-six thousand.
58. Seven hundred eight thousand two hundred.
59. Four hundred thousand eighty.

60. Two hundred thousand sixty-six.
61. One hundred nine thousand thirty-eight.
62. One hundred sixty thousand and six.
63. Eight hundred nine thousand ninety.
64. Seven hundred six thousand thirty.
65. Thirty thousand thirty.
66. Three thousand and three.
67. Ninety-five thousand two hundred six.
68. Six hundred twenty thousand forty-four.
69. One hundred one thousand and one.

LESSON III.

THREE PERIODS—UNITS, THOUSANDS, AND MILLIONS.

ORAL EXERCISES.

ART. 9. When a number is expressed by seven or more figures, the seventh figure denotes *millions;* the eighth figure, *ten-millions;* and the ninth figure, *hundred-millions.*

In reading a number expressed by eight figures, the eighth and seventh figures are read together as millions. Thus, 45,050,050 is read *forty-five million fifty thousand fifty.*

Read the following numbers;

1.	2.	3.	4.
30,000,000	37,300,450	75,000,740	90,090,090
45,000,000	40,207,006	80,000,008	83,830,083
7,000,000	50,350,042	71,017,049	46,060,706
27,000,000	8,080,080	65,560,000	66,666,666

ART. 10. In reading a number expressed by nine figures, the ninth, eighth, and seventh figures are read together as millions. Thus, 450,000,500 is read *four hundred fifty millions five hundred.*

Read the following numbers;

5.	6.	7.	8.
200,000,000	250,200,200	609,070,700	6,006,006
500,000,000	405,360,000	137,030,304	45,045,045
700,000,000	546,308,400	906,089,080	60,060,060
900,000,000	730,070,007	500,500,500	320,320,320

ART. 11. The third group of figures, expressing millions, ten-millions, and hundred-millions, constitutes the third or *Millions' Period.*

WRITTEN EXERCISES.

9. Write in words: 4,400,300; 40,050,300.
10. Write in words: 65,065,020; 300,070,600.
11. Write in words: 406,650,000; 650,065,008.

Express the following in figures as one number;

12. 65 millions 400 thousands 60 units.
13. 4 millions 752 thousands 458 units.
14. 30 millions 10 thousands 101 units.
15. 90 millions 9 thousands 70 units.
16. 683 millions 417 thousands 998 units.

Express the following in figures;

17. Fifty million thirty-two thousand six hundred forty.
18. Thirty-five million nine thousand two hundred six.
19. Five million five thousand and five.
20. One million one hundred thousand and ten.
21. Three hundred million seven hundred thirty.
22. Sixty-two million three hundred thousand.
23. Five hundred million five thousand.
24. Eight hundred million eight hundred.
25. Two million ten thousand eighty.

26. Four hundred million forty thousand four hundred and four.

27. Eighty-six million ten thousand fifty-six.

28. Seven hundred million seven thousand seven hundred twenty-five.

LESSON IV.

DEFINITIONS, PRINCIPLES, AND RULES.

To TEACHERS.—All definitions, principles, and rules should, when practicable, be presented concretely and illustrated before pupils are required to state them in general terms. See "Manual of Arithmetic."

ART. 12. A **Unit** is one.

A **Number** is a unit, or a collection of units.

An **Integer** is a whole number; as, 12, 75.

Arithmetic treats of numbers and their use.

ART. 13. There are three methods of expressing numbers:

1. By *words;* as, five, fifty, etc.
2. By *letters*, called the *Roman* method.
3. By *figures*, called the *Arabic* method.

ART. 14. **Notation** is the art of writing numbers.

Numeration is the art of reading numbers.

The term Notation is commonly used to denote the art of writing numbers by means of figures, called the *Arabic* Notation; and Numeration, to denote the art of reading numbers expressed by figures. The writing of numbers by means of letters is the *Roman* Notation (Art. 21).

ART. 15. A **Figure** is a character used to express number.

There are ten figures, viz: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The first (0) is called *Naught* or *Cipher*, and is used *to fill vacant* orders.

The other nine figures express each one or more units, and are called *Significant Figures.* They are also called *Digits.*

ART. 16. The successive figures which express a number, denote successive *Orders of Units.* These orders are numbered from the right; as, first, second, third, fourth, fifth, and so on.

A figure in the first place denotes units of the *first order;* in the second place, units of the *second order;* in the third place, units of the *third order*, and so on—the term *units* being used to express *ones* of any order.

ART. 17. Ten units equal one ten, ten tens equal one hundred, ten hundreds equal one thousand; and, generally, *ten units of any order equal one unit of the next higher order.*

NOTE.—The teacher can make this principle plain to the pupils by means of the numeral frame. It is easily shown that 10 ones or units equal 1 ten, and that 10 tens equal 1 hundred. *The successive figures which express a number are written on a scale of tens.*

ART. 18. The figures denoting the successive orders of units, are divided into groups of three figures each, called *Periods.*

The three orders of any period, counting from the right, denote, respectively, *units*, *tens*, and *hundreds* of that period.

The first three orders express units, tens, and hundreds of *units;* the second three orders, units, tens, and hundreds of *thousands;* the third three orders, units, tens, and hundreds of *millions*, etc.

ART. 19. The several orders may be named more briefly by calling the first order of each period by the

name of the period, and omitting the word "of" after tens and hundreds, thus:

Hundred-billions.	Ten-billions.	Billions.	,	Hundred-millions	Ten-millions.	Millions.	,	Hundred-thousands.	Ten-thousands.	Thousands.	,	Hundreds.	Tens.	Units.
4	4	4	,	3	3	3	,	2	2	2	,	1	1	1
4th Period. Billions.				3d Period. Millions.				2d Period. Thousands.				1st Period. Units.		

NOTE.—The fourth period is called *Billions;* the fifth, *Trillions;* the sixth, *Quadrillions;* the seventh, *Quintillions*, etc. There is no example in this work involving numbers expressed by more than three periods.

ART. 20. The **Value** of a figure is the number which it expresses. Hence,

The value of a figure depends upon the *order* in which it stands.

NOTE.—The distinction between the value of a figure and the figure itself is the same as the distinction between a number and the figure or figures which express it. A figure is not a number.

The value of each of the successive figures which express a number is a *Term.* The terms of 325 are 3 *hundreds*, 2 *tens*, 5 *units.*

Rule for Notation.—*Begin at the left, and write the figures of each period in their proper orders, filling all vacant orders and periods with ciphers.*

Rule for Numeration.—1. *Begin at the right, and separate the number into periods of three figures each.*

2. *Begin at the left, and read each period containing one or more significant figures as if it stood alone, adding its name.*

NOTES.—1. The name of the units' period is usually omitted.

2. In reading numbers, it is not necessary to connect the terms of a period or the successive periods with "and." 405,020 may be read *four hundred five thousand twenty.* The use of "and" is not, however, incorrect. This number may be read *four hundred and five thousand and twenty.* The latter reading is in accordance with general usage.

LESSON V.

ROMAN NOTATION.

ART. 21. In the Roman Notation, numbers are expressed by means of seven *capital letters*, viz: I, V, X, L, C, D, M.

I represents one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand.

ART. 22. All other numbers are expressed by repeating or combining these letters.

(1) When a letter is repeated, its value is repeated; thus: II represents 2; XX, 20; CCC, 300, etc.

(2) When a letter is placed before one of greater value, the less value is taken from the greater; thus: IV represents 4; IX, 9; XC, 90.

(3) When a letter is placed after one of greater value, the less value is added to the greater; thus: VI represents 6; XI, 11; CX, 110.

ART. 23. In the following table, numbers are expressed by letters and figures:

I, 1;	VIII, 8;	XV, 15;	XL, 40;
II, 2;	IX, 9;	XVI, 16;	L, 50;
III, 3;	X, 10;	XVII, 17;	LX, 60;
IV, 4;	XI, 11;	XVIII, 18;	LXX, 70;
V, 5;	XII, 12;	XIX, 19;	LXXX, 80;
VI, 6;	XIII, 13;	XX, 20;	XC, 90;
VII, 7;	XIV, 14;	XXX, 30;	C, 100.

WRITTEN EXERCISES.

Express the following numbers by figures:

1.	2.	3.
XIV	CCL	MDCL
XXIV	DCXC	MDLX
XXXIX	CCXC	MDLIX
XCVI	DCCL	MDCCC
CXI	DCLIX	MDCCCLX
CIX	MCCL	MDCCCLXXXIII

Express the following numbers by letters:

4.	5.	6.	7.
45	156	210	1500
76	184	550	1609
90	345	700	1808
93	433	750	1868
99	555	880	1940

Express the following numbers by letters:

8.	9.	10.	11.
204	1200	1685	2000
409	1350	1944	2050
540	1408	1865	2550
675	1590	1909	3010

LESSON VI.

QUESTIONS FOR REVIEW.

What is a unit? What is a number? What is an integer? Of what does arithmetic treat?

In how many ways may numbers be expressed? How are numbers expressed in the Arabic method? In the Roman method? What is notation? What is numeration?

What is a figure? How many different figures are used to

express numbers? Which are called significant figures? Which figure has no numerical value? What is its use?

What is meant by orders of units? How are the orders numbered? How many units of any order make one unit of the next higher order?

How many orders make a period? What are the names of these orders? Give the names of the first four periods.

What is the value of a figure? On what does the value of a figure depend?

Give the rule for notation. Give the rule for numeration.

How are numbers expressed in the Roman notation? Name the letters used, and give the value of each. How are numbers expressed by these letters?

What is the largest number that can be expressed by one figure? By two figures? By three figures?

What is the smallest whole number that can be expressed by one figure? By two figures? By three figures? By four figures? By seven figures? (Ans. to last, 1,000,000.)

ADDITION.

LESSON VII.

ORAL EXERCISES.

1. How many are 11 and 9? 11 and 10? 11 and 12? 11 and 14? 11 and 16? 11 and 18?

2. How many are 12 and 10? 12 and 12? 12 and 13? 12 and 16? 12 and 18? 12 and 15?

3. How many are 13 and 13? 13 and 12? 13 and 15? 13 and 14? 13 and 16? 13 and 17?

4. How many are 14 and 14? 14 and 10? 14 and 12? 14 and 11? 14 and 16? 14 and 15?

5. How many are 15 and 15? 15 and 10? 15 and 11? 15 and 13? 15 and 12? 15 and 14?

6. How many are 16 and 16? 17 and 17? 18 and 18? 19 and 19? 20 and 20? 25 and 25?

7. How many are 47 and 9 and 6 and 7 and 3?

8. How many are $64 + 8 + 7 + 6 + 9$?

9. $38 + 6 + 9 + 8 + 6 + 5 =$ how many?
10. $26 + 9 + 5 + 6 + 8 + 7 + 3 + 4 + 6 =$ how many?
11. Add by 2's from 17 to 41; 36 to 80.
12. Add by 3's from 2 to 50; from 24 to 72.
13. Add by 4's from 2 to 50; 27 to 87.
14. Add by 5's from 3 to 53; 37 to 87.
15. Add by 6's from 7 to 55; 34 to 94.
16. Add by 7's from 5 to 54; 54 to 96.
17. Add by 8's from 25 to 65; 47 to 95.
18. Add by 9's from 4 to 49; 49 to 94.
19. Add by 10's from 20 to 90; 17 to 87.
20. Add by 12's from 12 to 72.

WRITTEN EXERCISES.

21. What is the sum of 121, 233, 123, 332, 231, 323, 123, 232, and 333?

PROCESS.

121	Write the numbers so that the units shall form the first column; the tens, the second column; and the hundreds, the third column.
233	Begin with the units' column, and add, naming results only, thus: 5, 8, 11, 12, 14, 17, 20, 21,—21 units, equal 2 tens and 1 unit.
123	
332	
231	Write the 1 unit under the units' column, and add the 2 tens with the tens' column, thus: 5, 8, 10, 12, 15, 18, 20, 23, 25,—25 tens, equal 2 hundreds and 5 tens.
323	
123	
232	Write the 5 tens under the tens' column, and add the 2 hundreds with the hundreds' column, thus: 5, 7, 8, 11, 13, 16, 17, 19, 20,—20 hundreds, equal 2 thousands and 0 hundreds.
333	
2051, *Sum.*	
21	Write the 0 hundreds under the hundreds' column, and write the 2 thousands in thousands' place. The sum is 2051.
23	
18	
2051, *Sum.*	To test the accuracy of the work, add the columns downward.

NOTE.—The manner in which the sums of the several columns are united mentally is shown below the double lines.

Copy and add the following numbers;

22.	23.	24.	25.	26.
33232	15215	23512	52134	24137
32323	14343	30425	34445	16126
23213	45046	41341	53054	20050
13221	50350	23301	44052	16654
32233	33432	41545	25253	33456
232111	43543	43453	34545	44162
323212	23343	25445	41534	23206
232021	45452	41505	22335	36562

27.	28.	29.	30.	31.
43260	35260	305129	106406	240455
32345	16165	224603	140757	140364
16606	32542	350164	265075	255074
46060	36344	255234	160214	463647
50050	24030	145344	344123	545633
16566	33246	242456	534522	160572
24656	21438	145346	261231	720277
32562	45546	200500	160214	477353
45672	37408	560347	754321	175324

32.	33.	34.	35.	36.
32620	7121365	19864	42764	5784062
75437	2171634	34687	8738768	309984
50743	1237773	46768	634187	67792
64017	7143656	65837	9463506	9999
32516	2674467	80040	324483	999807
18416	6734765	18608	43832	9095742
13673	6574636	36084	2741608	3407856
31654	7147347	45687	837860	9900789
46810	8342108	38762	708078	444444
53472	3562735	77777	666666	666666
16172	4444444	88888	555555	99999

37.	38.	39.	40.
$27150	$ 28165	$4009100	$799038
45263	41412	270036	691104
58016	175550	116023	136585
12141	250600	1085075	117760
95700	182737	2063	85891
855	457019	47184	570897
3038	6118	490737	464109
63025	3027	1927808	155025
$	$	$	$

NOTE.—In writing sums of money to add or subtract, the dollar sign is written but twice; viz, before the first number and before the answer.

41. Add $13145, $9187, $10050, $1087, $96050, $75187, $55475, $250084, and $16025.

42. Add $72010, $96819, $3118, $10050, $39142, $83115, $790068, $1600, and $45.

43. Add $260, $372, $13687, $45860, $45083, $90850, $11283, and $1805.

44. What is the sum of four hundred four; four thousand forty; forty thousand four hundred; and four million four hundred thousand?

45. Add thirty-six thousand three hundred twenty-five; fourteen thousand forty-six; twenty-three thousand four hundred five; fifteen thousand sixteen; and three hundred six thousand three hundred four.

46. What is the sum of three million one thousand fifty-six; six hundred thousand six hundred twenty-five; four million forty-two thousand four; and forty-five million six hundred fifty thousand?

47. What is the sum of sixteen million four thousand sixty-five; three hundred thousand two hundred fifty-six; seven thousand forty; five million five thousand seven; four hundred thousand six hundred seven; and *three hundred* forty thousand seventy?

48. What is the sum of forty-five million seven thousand seventy; six million sixty-five thousand two hundred six; seventy-five thousand forty-four; eight million eight thousand eight; eighteen million eighteen thousand eighteen; and eighty million eight hundred thousand?

LESSON VIII.

ORAL PROBLEMS.

1. Frank has 5 marbles in one hand and 2 marbles in the other: how many has he in both hands?

SOLUTION.—5 marbles and 2 marbles are 7 marbles: Frank has 7 marbles in both hands.

2. A drover bought 9 sheep of one farmer and 2 sheep of another: how many sheep did he buy?

3. A grocer sold 8 pounds of sugar to one customer, 3 pounds to another, and 2 pounds to another: how many pounds of sugar did he sell?

4. A man walked 4 miles the first hour, 3 miles the second, and 2 miles the third: how many miles did he walk in the 3 hours?

5. A man gave $26 for a coat and $5 for a hat: how many dollars did he give for both?

6. A drover bought 19 cows of one man, 4 of another, and 5 of another: how many cows did he buy?

7. James picked 27 peaches from one limb and 5 peaches from another: how many peaches did he pick from both limbs?

8. Mary has written 16 lines: if she write 5 lines more, how many lines will she then have written?

9. George gave 15 cents for a slate and 5 cents for a pencil: how many cents did he give for both?

10. John solved 18 problems before school and 6 problems in school: how many problems did he solve?

11. A farmer bought a cow for $27 and a calf for $6: how much did he pay for both?

12. The head of a fish is 5 inches long, its body 16 inches, and its tail 6 inches: how long is the fish?

13. In a certain orchard there are 29 apple-trees, 5 pear-trees, and 6 peach-trees: how many trees in the orchard?

14. William gave a blind boy 19 cents, John gave him 15 cents, and Charles 6 cents: how many cents did they all give him?

15. A man bought a set of harness for $37, a saddle for $7, and a bridle for $4: how much did he pay for all?

16. Frank gave 10 cents for a lead-pencil, 5 cents for a piece of rubber, and 7 cents for paper: how much did the three articles cost?

17. A gentleman gave $36 for a suit of clothes, $7 for a pair of boots, and $5 for a hat: how much did he pay for all?

18. A farmer raised 16 loads of hay in one field, 8 loads in another, and 7 loads in another: how much hay did he raise?

19. A lady paid $27 for a shawl, $8 for a bonnet, and $4 for a pair of shoes: how much did she pay for all?

20. A merchant sold 18 yards of muslin to one customer, 7 yards to another, and 8 yards to another: how many yards did he sell?

21. A man paid $23 for a coat, $9 for a pair of pants, and $6 for a vest: how much did he pay for the suit?

22. A lady gave 15 cents for thread, 8 cents for needles, and 7 cents for pins: how many cents did she spend?

23. A gentleman gave $95 for a horse, $15 for a saddle, and $5 for a bridle: how much did he pay for all?

WRITTEN PROBLEMS.

24. July has 31 days; August, 31; September, 30; October, 31; November, 30; and December, 31: how many days in the last six months of the year?

25. January has 31 days; February (except in leap year), 28; March, 31; April, 30; May, 31; and June, 30: how many days in the first six months of the year?

26. A man bought four loads of hay, the first weighing 2130 pounds, the second 2312 pounds, the third 2232 pounds, and the fourth 2322 pounds: how many pounds of hay in the four loads?

27. A grain dealer bought 2350 bushels of wheat on Monday, 4215 bushels on Tuesday, 3245 bushels on Wednesday, 1500 bushels on Thursday, 2424 bushels on Friday, and 1350 bushels on Saturday: how many bushels did he buy?

28. In a city having five wards, there are 345 voters in the first ward, 443 in the second, 213 in the third, 523 in the fourth, and 425 in the fifth: how many voters in the city?

29. The first ward of a city contains 1675 youth of school age; the second, 2357 youth; the third, 2347; the fourth, 3270; and the fifth, 2677: how many youth of school age in the city?

30. A father gave to his eldest son 225 acres of land; to the second, 155 acres; to the third, 145 acres; and to the youngest, 124 acres: how many acres did he give to all?

31. The first three cars of a freight train contain 35240 pounds each; the next four cars, 25345 pounds each; the next two cars, 31540 pounds each; and the last car, 25432 pounds: how many pounds of freight in the ten cars?

32. A gentleman owns 5 farms, containing respectively 285 acres, 345 acres, 146 acres, 438 acres, and 248 acres: how many acres of land does he own?

33. A steam-ship sailed 217 miles the first day; 265 miles the second; 227 miles the third; 187 miles the fourth; and 168 miles the fifth: how many miles did it sail in the five days?

34. The distance by railroad from Boston to Springfield is 98 miles; from Springfield to Albany, 103 miles; from Albany to Buffalo, 298 miles; from Buffalo to Cleveland, 183 miles; from Cleveland to Chicago, 355 miles: how far from Boston to Chicago?

35. A man paid $3575 for a lot, $5460 for a house, $875 for a stable, and $675 for other improvements: what did the property cost?

36. A merchant's sales on Monday, were $275; on Tuesday, $368; on Wednesday, $540; on Thursday, $165; on Friday, $398; and on Saturday, $754: what was the amount of his sales in the week?

37. Ohio contains 41060 square miles; Indiana, 36350 square miles; Illinois, 56650 square miles; and Michigan, 58915 square miles: what is the area in square miles of these four states?

38. Maine contains 33040 square miles; New Hampshire, 9305; Vermont, 9565; Massachusetts, 8315; Rhode Island, 1250; and Connecticut, 4990: how many square miles in these six New England states?

39. The population of New York, in 1880, was 5082871; of Pennsylvania, 4282891; of Ohio, 3198062; of Illinois, 3077871; of Missouri, 2168380; and of Indiana, 1978301: what was the population of these six states?

40. The population of Alabama, in 1880, was 1262505; Georgia, 1542180; Kentucky, 1648690; Mississippi, 1131597; North Carolina, 1399750; South Carolina, 995577; Texas, 1591749; and Virginia, 1512565: what was the population of these eight states?

LESSON IX.

DEFINITIONS AND RULE.

ART. 24. The **Sum** of two or more numbers contains as many units as all the numbers taken together. It is also called the *Amount*.

ART. 25. **Addition** is the process of finding the sum of two or more numbers.

ART. 26. The **Sign of Addition** is +. It is called *plus*, meaning *more*.

When placed between two numbers, it shows that they are to be added. Thus, $8 + 5$ is read 8 *plus* 5, and it shows that 5 is to be added to 8.

The **Sign of Equality** is =. It is read *equals*, or *is equal to*. Thus, $7 + 8 = 15$ is read 7 *plus* 8 *equals* 15.

ART. 27. Numbers are either *Concrete* or *Abstract*.

A **Concrete Number** is applied to a particular thing; as, 4 pears, 7 hours, 30 steps.

An **Abstract Number** is not applied to any particular thing; as, 4, 7, 30.

ART. 28. Fourteen balls and 12 balls are numbers of the *same kind*, and 6 tens and 3 tens are numbers of the *same order*. Numbers of the same kind or order are *Like Numbers*.

Only like numbers can be added. Three pears and 4 peaches can not be added, nor can 3 units and 4 tens be added, except by first reducing the 3 tens to 30 units.

Rule for Addition.—1. *Write the numbers to be added so that figures denoting units of the same order shall be in the same column, and draw a line underneath.*

2. *Beginning with units, add each column, and write the sum, when less than ten, underneath.*

3. *When the sum of any column exceeds nine, write the right-hand figure under the column added, and add the number denoted by the left-hand figure or figures with the next column.*

4. *Write the entire sum of the left-hand column.*

Proof.—*Add the columns downward.*

SUBTRACTION.

LESSON X.

ORAL EXERCISES.

1. How many is 11 less 3? 12 less 3? 21 less 3? 31 less 3? 22 less 3? 32 less 3?

2. How many is 13 less 4? 23 less 4? 43 less 4? 13 less 5? 33 less 5? 53 less 5? 63 less 5?

3. How many is 13 less 6? 33 less 6? 63 less 6? 14 less 6? 44 less 6? 54 less 6?

4. How many is 12 less 7? 32 less 7? 52 less 7? 62 less 7? 15 less 7? 45 less 7? 55 less 7?

5. How many is 15 less 8? 25 less 8? 45 less 8? 35 less 8? 65 less 8? 75 less 8? 95 less 8?

6. How many is 17 less 8? 37 less 8? 57 less 8? 47 less 8? 67 less 8? 77 less 8? 97 less 8?

7. How many is 16 less 9? 36 less 9? 46 less 9? 26 less 9? 66 less 9? 86 less 9? 76 less 9?

8. How many is 13 less 9? 33 less 9? 43 less 9? 63 less 9? 53 less 9? 73 less 9? 83 less 9?

9. How many is 23 less 11? 24 less 12? 25 less 12? 22 less 12? 25 less 13? 24 less 13? 26 less 13?

10. How many is 25 less 14? 28 less 14? 30 less 15? 25 less 15? 26 less 15? 27 less 15? 42 less 20?

11. *Subtract* by 2's from 50 back to 0; 41 back to 1.

12. Subtract by 3's from 70 back to 31; 41 to 2.
13. Subtract by 4's from 42 back to 2; 51 to 3.
14. Subtract by 5's from 53 back to 3; 62 to 22.
15. Subtract by 6's from 52 back to 4; 63 to 15.
16. Subtract by 7's from 54 back to 5; 73 to 17.
17. Subtract by 8's from 68 back to 4; 75 to 35.
18. Subtract by 9's from 48 back to 3; 75 to 30.
19. Subtract by 10's from 57 to 7; 68 to 18.
20. Subtract by 12's from 48 to 0; 84 to 48.

WRITTEN EXERCISES.

21. From 358 take 235; from 3683 take 2542.
22. From 5334 take 2726.

Write units under units, tens under tens, etc.

PROCESS.

Minuend,	5334
Subtrahend,	2726
Difference,	2608

Since 6 units can not be taken from 4 units, add 10 units to the 4 units, making 14 units, and take 6 units from the 14 units, and write 8 units (the difference) below. To balance the 10 units added to the minuend, add 1 ten (equal to 10 *units*) to the 2 tens, making 3 tens, and take 3 tens from 3 tens, and write 0 (the difference) below.

Since 7 hundreds can not be taken from 3 hundreds, add 10 hundreds to the 3 hundreds, making 13 hundreds, and take 7 hundreds from 13 hundreds, and write 6 hundreds (the difference) below. To balance the 10 hundreds added to the minuend, add 1 thousand (equal to 10 *hundreds*) to the 2 thousands, making 3 thousands, and take 3 thousands from 5 thousands, and write 2 thousands (the difference) below. The difference is 2608.

NOTES.—1. The teacher should show that the adding of 10 to a term of the minuend and 1 to the next higher term of the subtrahend increases both minuend and subtrahend *equally*, and, hence, does not affect the difference. The 10 is not "borrowed."

2. Instead of adding 1 to the next term of the subtrahend, 1 may be taken from the next term of the minuend. Many teachers prefer the latter method. The 10 need not be obtained by reducing 1 of the next higher order. The 10 added to a term of the minuend is balanced by taking 1 from the next higher term.

	23.	**24.**	**25.**	**26.**	**27.**
From	463	3272	1385	5754	3416
Take	336	2147	1276	3457	507

	28.	**29.**	**30.**	**31.**	**32.**
From	3041	14406	20670	30401	67113
Take	2637	7345	17356	20576	8094

	33.	**34.**	**35.**	**36.**
From	$3075	$70563	$50000	$30406
Take	1847	29487	9045	7085

37. From sixteen thousand twenty-six take seven thousand five hundred forty-five.

38. From five hundred seven thousand two hundred two take eighty-seven thousand sixty-seven.

39. From forty million seven hundred take eighty thousand eighty-eight.

40. From fifty-seven million six thousand sixty take eight million eight thousand and eight.

LESSON XI.

ORAL PROBLEMS.

1. Charles earned 21 cents by selling papers, and gave 4 cents for a comb: how many cents had he left?

SOLUTION.—21 cents less 4 cents are 17 cents; Charles had 17 cents left.

2. Kate is 15 years old and her sister is 6 years younger: what is her sister's age?

3. There are 21 passengers in a car: if 7 of them leave at a station, how many will remain?

4. There are 13 men in one coach and 8 men in another: how many men in the first coach more than in the second?

5. A man gave $12 for a saddle and $4 for a bridle: how much did the saddle cost more than the bridle?

6. Henry wrote 21 words, but misspelled 5 of them: how many words did he spell correctly?

7. Charles's lesson consists of 15 examples, and he has solved 9 of them: how many has he not solved?

8. James is 14 years old, and his brother Henry is 5 years younger: how old is Henry?

9. A school has enrolled 65 pupils, and 8 pupils are absent: how many are present?

10. Mr. Smith is 44 years of age and his youngest son is 8 years of age: what is the difference in their ages?

11. A school contains 9 more girls than boys: if there are 56 girls, what is the number of boys?

12. From a cask containing 45 gallons of molasses, 39 gallons were sold: how many gallons remained unsold?

13. In a school, 63 pupils are enrolled and 54 are present: how many pupils are absent?

14. If a man earn $45 a month, and spend $36, how much does he lay up?

15. A man gave $35 for a watch and $12 for a chain: how much did the watch cost more than the chain?

16. Charles has 17 marbles and John 8: how many more marbles has Charles than John?

17. A teacher asked his class 52 questions, and 8 questions were answered incorrectly: how many were answered correctly?

WRITTEN PROBLEMS.

18. A grocer bought 585 pounds of sugar and sold 231 pounds: how many pounds had he left?

19. A farm contains 358 acres of land: if 175 acres should be sold, how many acres would be left?

20. A man bought a house for $4370, and sold it for $6450: how much did he gain?

21. A man bought 3487 bushels of wheat, and sold 1495 bushels: how many bushels had he left?

22. The number of youth of school age in a certain city is 1234, and 756 pupils are enrolled in the schools: how many youth are not enrolled?

23. A man whose income is $1850, expends annually $1365: how much does he lay up?

24. A merchant having $11315 in bank, drew out $976: how much remained in the bank?

25. The Pilgrims landed at Plymouth in 1620, and our National Independence was declared in 1776: how many years between the two events?

26. The first steam-boat was made in 1807, and the Atlantic Cable was laid in 1866: how many years between the two events?

27. America was discovered in 1492, and the Pilgrims landed at Plymouth in 1620: how many years intervened?

28. Mt. Etna is 10874 feet high, and Mt. Vesuvius 3948 feet: how much higher is Etna than Vesuvius?

29. Mont Blanc, in Europe, is 15668 feet high, and Mount Sorata, in South America, is 21286 feet high: what is the difference in their height?

30. An army of 30340 men lost 7568 in battle: how many men did it then contain?

31. The area of the six New England states is 66,465 square miles, and the area of California is 158,360 square miles: how much greater in area is California than New England?

32. The population of the United States, in 1870, was 38,115,641, and in 1880 the population was 50,155,783: *what was the increase* in ten years?

LESSON XII.

DEFINITIONS AND RULE.

ART. 29. **Subtraction** is the process of taking a less number from a greater.

The **Minuend** is the number from which the less number is taken.

The **Subtrahend** is the number taken from the minuend.

The **Difference** is the number obtained by subtracting.

The difference is the number of units in the minuend more than in the subtrahend.

ART. 30. The **Sign of Subtraction** is —. It is read *minus* or *less*. It shows that the number after it is to be subtracted from the number before it.

Only like numbers can be subtracted. Three pencils can not be subtracted from 7 books, nor 3 units from 7 tens.

Rule for Subtraction.—1. *Write the subtrahend under the minuend, placing units under units, tens under tens, hundreds under hundreds, etc.*

2. *Begin at the right, and subtract each term of the subtrahend from the term above it, and write the difference underneath.*

3. *When any term of the subtrahend is greater than the term above it, add 10 to the upper term, and then subtract, and write the difference underneath.*

4. *When 10 has been added to the upper term, add 1 to the next higher term of the subtrahend before subtracting.*

Proof.— *Add the remainder and subtrahend; if their sum is equal to the minuend, the work is correct.*

NOTE.—Instead of adding 1 to the next term of the subtrahend, 1 may be subtracted from the next term of the minuend.

LESSON XIII.

PROBLEMS COMBINING ADDITION AND SUBTRACTION.

ORAL PROBLEMS.

1. Robert picked 21 peaches, and gave 7 to his sister and 8 to his brother: how many peaches had he left?

SOLUTION.—7 peaches and 8 peaches are 15 peaches; 21 peaches less 15 peaches are 6 peaches: he had 6 peaches left.

2. An orchard contains 33 trees, and 17 of them are apple-trees, 8 peach-trees, and the rest pear-trees: how many pear-trees in the orchard?

3. A grocer bought 35 bushels of apples, and sold 17 bushels to A, 9 bushels to B, and the rest to C: how many bushels did he sell to C?

4. A man earned $45, and paid $15 for house rent, $8 for flour, $7 for shoes, and $10 for groceries: how much had he left?

5. A man sees 15 pigeons on one branch of a tree, and 9 pigeons on another branch: if 7 should fly away, how many would be left on the tree?

6. A farmer had 23 chickens, but 7 of them died and 5 were carried off by a hawk: how many chickens had he left?

7. A drover bought 17 sheep of one farmer, 6 sheep of another, and 8 of another, and then sold 9 of them to a butcher: how many sheep had he left?

8. A farmer bought a cow for $25, a calf for $8, and a sheep for $5, and gave in payment a colt worth $30 and the balance in money: how much money did he pay?

9. A lad earned 50 cents by selling papers, and paid 25 cents for a reader, 8 cents for a slate, and 12 cents for a copy-book: how many cents had he left?

10. From the sum of 17, 8, and 6, take 9.

WRITTEN PROBLEMS.

11. From a piece of carpeting containing 150 yards, a merchant sold 3 carpets, containing 27, 39, and 42 yards, respectively: how many yards did he sell? How many yards were left?

12. From the sum of \$4750, \$2284, \$960, and \$435 take \$5863.

13. A regiment entered the service with 1088 men; 150 were killed in battle, 65 died of disease, 24 deserted, and 250 were discharged: how many remained?

14. A grain dealer bought 1250 bushels of wheat on Monday, 2145 bushels on Tuesday, and 3240 bushels on Wednesday, and on Thursday he sold 5450 bushels: how many bushels did he buy? How many bushels had he left?

15. A man deposited \$175, \$141, \$75, \$304, and \$250 in a bank, and then drew out \$480 and \$225: how many dollars did he deposit? How many did he draw out? How many remained in the bank?

16. A farmer raised 1480 bushels of wheat, and sold 580 bushels to one man and 475 bushels to another: how many bushels were left?

17. Rhode Island contains 1250 square miles; Delaware, 2055 square miles; Connecticut, 4990; New Jersey, 7815; and Kentucky, 40400: how many more square miles in Kentucky than in the other four states?

18. From the sum of 2368 and 1299 take their difference.

19. From the sum of 2348 and 1864 take their difference.

20. From the sum of 506703 and 340067 take their difference.

21. From the sum of \$3040, \$2685, and \$872, take the sum of \$5231 and \$165.

MULTIPLICATION.

LESSON XIV.

ORAL EXERCISES.

1. How many are 4 times 3? 6 times 3? 8 times 3? 10 times 3? 5 times 3? 7 times 3? 9 times 3?

2. How many are 4 times 4? 6 times 4? 8 times 4? 10 times 4? 5 times 4? 7 times 4? 9 times 4?

3. How many are 3 times 5? 5 times 5? 7 times 5? 9 times 5? 6 times 5? 8 times 5? 10 times 5?

4. How many are 5 times 6? 7 times 6? 9 times 6? 4 times 6? 6 times 6? 8 times 6? 10 times 6?

5. How many are 3 times 7? 5 times 7? 7 times 7? 9 times 7? 4 times 7? 6 times 7? 8 times 7? 10 times 7?

6. How many are 4 times 8? 6 times 8? 8 times 8? 10 times 8? 5 times 8? 7 times 8? 9 times 8?

7. How many are 3 times 9? 5 times 9? 7 times 9? 9 times 9? 6 times 9? 8 times 9? 10 times 9?

8. How many are 5 times 10? 7 times 10? 8 times 10? 10 times 10? 11 times 10? 12 times 10?

9. How many are 4 times 11? 6 times 11? 8 times 11? 5 times 11? 7 times 11? 9 times 11?

10. How many are 3 times 12? 5 times 12? 7 times 12? 9 times 12? 11 times 12? 6 times 12? 8 times 12? 10 times 12? 12 times 12?

WRITTEN EXERCISES.

11. Multiply 345 by 6.

12. Multiply 435 by 7; by 8; by 6; by 5; by 9.

13. Multiply 5674 by 5; by 7; *by 8; by 6;* by 9.

PROCESS.

Multiplicand,	345
Multiplier,	6
Product,	2070

14. Multiply 6708 by 6; by 5; by 7; by 9; by 8.

15. Multiply 8099 by 5; by 6; by 7; by 8; by 9.

16. Multiply 2706 by 6, and the product by 8.

17. Multiply 4560 by 7, and the product by 9, and this product by 5.

18. Multiply 3048 by 8, and the product by 7, and this product by 6.

19. Multiply 456 by 43.

Write the multiplier under the multiplicand, placing units under units and tens under tens.

First multiply by the 3 units (as in the preceding examples), which gives 1368 for the *first* partial product.

Next multiply by the 4 tens, and since units multiplied by tens (or tens by units) produce *tens*, tens by tens *hundreds*, and hundreds by tens *thousands*, the *second* partial product, is 4 *tens*, 2 *hundreds*, 8 *thousands*, and 1 *ten thousand*, which are written in their proper orders.

PROCESS.

Multiplicand,	456
Multiplier,	43
Partial products,	1368
	1824
Product,	19608

ILLUSTRATIVE PROCESS.

$456 \times 3 =$	1368
$456 \times 40 =$	18240
Product,	19608

Add the two partial products, and their sum, which is 19608, is the product required.

NOTE.—The teacher should show that units multiplied by tens produce *tens;* tens by tens, *hundreds;* hundreds by tens, *thousands*, etc. This may be done, in the above example, by changing the 4 tens into 40 units. 40 times 6 units = 240 units, or 24 *tens;* and 40 times 5 tens = 200 tens, or 20 *hundreds*, etc. The first figure of each partial product is written under the multiplier which produces it.

20. Multiply 4606 by 54; by 75; by 86; by 69.

21. Multiply 2327 by 36; by 65; by 78; by 84.

22. Multiply 30279 by 34; by 57; by 66; by 88.

23. Multiply 43065 by 41; by 61; by 50; by 80.

24. Multiply 34756 by 65; by 71; by 60; by 90.

25. Multiply 40732 by 55; by 66; by 88; by 77.

26. Multiply 81017 by 44; by 99; by 59; by 69.
27. Multiply 63006 by 73; by 37; by 78; by 87.
28. Multiply 4068 by 346.
29. Multiply 63082 by 435.
30. Multiply 40694 by 327.
31. Multiply 45053 by 644.
32. Multiply 343607 by 482.
33. Multiply 60563 by 2346.
34. Multiply 5378 by 2435.
35. Multiply 3254 by 425.
36. Multiply 2346 by 2346.
37. Multiply 4165 by 4165.

(28) PROCESS.

4068
346
24408
16272
12204
Product, 1407528

NOTE.—The product of a number by itself is the *square* of the number, and the product of a number by its square is the *cube* of the number.

38. Multiply 342 by 342, and the product by 342.
39. Multiply 453 by 453, and the product by 453.
40. Multiply 2745 by 306.

PROCESS.

2745
306
Partial products, 16470, 8235
Product, 839970

Multiply successively by the first and third terms of the multiplier, and since units multiplied by hundreds produce *hundreds*, write the first figure of the second partial product in hundreds' order. In 306 there are no tens to be used as a multiplier.

41. Multiply 4086 by 608; by 707; by 509.
42. Multiply 7908 by 506; by 2008; by 6005.
43. Multiply 6075 by 3008; by 5006; by 4009.
44. Multiply 3460 by 407; by 506; by 4008.
45. Multiply 4607 by 705; by 608; by 5006.
46. Multiply 5083 by 706; by 907; by 6005.
47. Multiply 45063 by 7008; by 6307.
48. Multiply 30876 by 3406; by 7009.
49. Multiply 14800 by 47; by 54; by 6065.
50. Multiply 348000 by 324; by 462; by 503.

51. Multiply 230800 by 405; by 666.
52. Multiply 23050 by 4006.
53. Multiply 2059 by 3400.
54. Multiply 4306 by 67000.
55. Multiply 5284 by 2900.
56. Multiply 4509 by 56000.
57. Multiply 6375 by 40800.
58. Multiply 92005 by 68000.
59. Multiply 89000 by 4500.
60. Multiply 94000 by 3600.
61. Multiply 90800 by 640.
62. Multiply 4760 by 203000.
63. Multiply 88000 by 8800.
64. Multiply 3074 by 1000.
65. Multiply 30840 by 5000.

(53) PROCESS.

2059
3400
8236
6177
7000600

(59) PROCESS.

89000
4500
445
356
400500000

66. Multiply $475 by 24; by 63; by 145.
67. Multiply $6025 by 48; by 65; by 407; by 606.
68. Multiply $36580 by 425; by 708; by 240; by 800.
69. Multiply $4016 by 24; by 9; by 250; by 700.

LESSON XV.

ORAL PROBLEMS.

1. If a man earn $3 a day, how many dollars will he earn in 6 days?

SOLUTION.—If a man earn $3 in one day, in 6 days he will earn 6 times $3, which is $18.

2. If a boy walk 3 miles a day in attending school, how many miles will he walk in 10 days?

3. There are 3 feet in a yard: how many feet in 2 yards? In 4 yards? 5 yards? 7 yards?

4. There are 4 quarts in a gallon: how many quarts in 5 gallons? In 7 gallons? 10 gallons?

5. There are 5 cents in a half-dime: how many cents in 3 half-dimes? In 5 half-dimes?

6. If there are 5 school-days in a week, how many school-days in 6 weeks? In 8 weeks? In 10 weeks?

7. If a horse travel 6 miles an hour, how far will it travel in 5 hours? In 10 hours?

8. There are 6 feet in a fathom: how many feet in 7 fathoms? In 9 fathoms? 8 fathoms?

9. There are 6 days for labor in each week: how many days for labor in 6 weeks? In 9 weeks?

10. There are 8 rows of trees in an orchard, and 6 trees in each row: how many trees in the orchard?

11. There are 4 pecks in a bushel: how many pecks in 6 bushels? In 8 bushels? 10 bushels? 12 bushels?

12. There are 8 quarts in 1 peck: how many quarts in 3 pecks? 5 pecks? 7 pecks? 6 pecks?

13. There are 8 pints in a gallon: how many pints in 4 gallons? 6 gallons? 8 gallons? 10 gallons?

14. If an orange is worth 5 apples, how many apples are 7 oranges worth? 10 oranges? 8 oranges?

15. James has 8 marbles, and John has 6 times as many: how many marbles has John?

16. If a man earn $8 a week, how much will he earn in 9 weeks? 11 weeks? 8 weeks?

17. A railroad car has 8 wheels; how many wheels has a train of 7 cars? A train of 9 cars?

18. If a horse eat 8 quarts of oats each day, how many quarts will he eat in 6 days? In 10 days?

19. Charles received $9 a month as errand-boy: how much will he earn in 10 months? In 12 months?

20. What will 7 lead-pencils cost, at 6 cents apiece?

21. What will 6 oranges cost, at 5 cents apiece?

22. If a pint of oil cost 8 cents, what will 8 pints cost? 10 pints? 12 pints?

23. What will 6 bananas cost at two for 5 cents?

24. What will 5 barrels of flour cost at $9 a barrel?

25. What will 8 pounds of beef cost at 10 cents a pound? At 12 cents? At 15 cents?

26. What will 6 quarts of strawberries cost at 8 cents a quart? At 10 cents? At 12 cents?

27. What will 12 heads of cabbage cost at 4 cents a head? At 5 cents? At 6 cents?

28. What will 8 boxes of matches cost at 15 cents a box? At 20 cents? At 25 cents?

29. What will 4 pairs of boots cost at $6 a pair?

30. What will 12 yards of muslin cost at 8 cents a yard? At 6 cents? At 10 cents?

31. What will 11 sheep cost at $7 a head?

32. What will 8 barrels of apples cost at $2 a barrel? At $3 a barrel?

WRITTEN PROBLEMS.

33. There are 320 rods in a mile: how many rods are there in 3 miles? In 12 miles? 25 miles?

34. If a train of cars run 425 miles a day, how far will it run in 8 days? In 45 days?

35. If 135 tons of iron rails will make one mile of railroad, how many tons will make 245 miles?

36. If a ship sail 216 miles a day, how far will it sail in 12 days? In 23 days? In 45 days?

37. If a web of flannel contain 46 yards, how many yards in 46 webs? 165 webs? 480 webs?

38. A father divided his estate between four sons, giving to each $3545: what was the value of the estate?

39. There are 60 minutes in an hour: how many minutes in 24 hours, or one day? How many minutes in 7 days, or a week?

40. Enos lived 905 years: how many days did he live, allowing 365 days to the year?

41. A planter raised 208 bales of cotton, each bale weighing 440 pounds: how many pounds of cotton did he raise?

42. If a garrison of soldiers consume 4865 pounds of bread a day, how many pounds will supply the garrison 48 days? 406 days? 504 days?

43. What will it cost to build 305 miles of railroad at $7525 a mile?

44. There are 5280 feet in a mile: how many feet in 80 miles? In 75 miles? 605 miles?

45. The earth moves in its orbit at an average rate of 68400 miles in an hour: how far does it move in 24 hours? In 48 hours? In 120 hours?

46. If a carriage-wheel revolve 280 times in running a mile, how many times will it revolve in running 18 miles? 75 miles? 250 miles?

47. A canal-boat was loaded with 245 bales of hay, weighing 280 pounds each: what was the weight of the cargo?

48. There are 480 sheets of paper in a ream: how many sheets are there in 60 reams? 560 reams?

49. If an acre of land produce 380 pounds of cotton, how many pounds will 248 acres produce?

50. A steam-boat made 145 trips in a season, and carried, on an average, 280 passengers each trip: how many passengers did she carry during the season?

51. There are 3600 seconds in one hour: how many seconds are there in 24 hours? In 168 hours?

52. Light moves 192000 miles in a second: how far does it move in 60 seconds, or one minute? In one hour? In 24 hours, or one day?

53. Sound moves 1090 feet in a second: how far will it move in 60 seconds, or one minute? In 3600 seconds, or one hour?

54. A ship has provisions enough to allow the crew 130 pounds a day for 90 days: how many pounds of provisions are aboard?

55. What will 1700 tons of railroad iron cost at $75 a ton? At $84 a ton? At $96 a ton?

56. A merchant sold 405 bales of cotton at $75 a bale: how much did he receive?

57. A farmer sold 90 tons of hay at $12 a ton: how much did he receive?

58. A drover bought 120 head of cattle at an average cost of $37 a head: how much did the drove cost?

59. What will 640 acres of land cost at $62 an acre? At $25 an acre? At $33 an acre?

60. A merchant bought 5 dozens of silver watches at $14 apiece: how much did they cost?

61. What will 240 barrels of flour cost at $6 a barrel? At $5 a barrel? At $8 a barrel?

62. An army is composed of 54 regiments, containing, on an average, 670 men each: how many men in the army?

63. If a steamer can run 260 miles a day, how far can it run in 10 days? In 100 days? In 75 days?

64. In a field of corn there are 70 rows, and each row has 280 hills, and each hill 3 stalks: how many stalks of corn in the field?

65. In a train of 37 cars, each car contains 9850 pounds of freight: how much freight in the train?

66. If 980 pounds of bread will supply the inmates of a state prison one day, how many pounds will supply them 365 days, or one year?

67. If a sack of salt contain 168 pounds, what will be the weight of 150 sacks? 1600 sacks?

68. A merchant bought 18 firkins of butter, each weighing 32 pounds, at 27 cents a pound: what did it cost?

69. A train of 27 cars is loaded with iron; each car contains 48 bars, and each bar weighs 365 pounds: what is the weight of the cargo?

70. A grocer bought 68 barrels of sugar, each weighing 275 pounds, at 8 cents a pound: how much did the sugar cost?

LESSON XVI.

DEFINITIONS, PRINCIPLES, AND RULES.

ART. 31. **Multiplication** is the process of taking one number as many times as there are units in another.

The **Multiplicand** is the number taken or multiplied.

The **Multiplier** is the number denoting how many times the multiplicand is taken.

The **Product** is the number obtained by multiplying.

The multiplicand and multiplier are *Factors* of the product.

ART. 32. The **Sign of Multiplication** is $\times$, and is read *multiplied by*. When placed between two numbers it shows that the number before it is to be multiplied by the number after it. Thus: 6×3 is read 6 *multiplied by* 3.

NOTE.—Since a change in the order of the factors does not change the product, the number after the sign may be considered the multiplicand, and the sign read *times*. 6×3 may be read 6 *times* 3.

ART. 33. Multiplication is a short method of finding the sum of several *equal* numbers. The product of 5×4 is the sum of $5 + 5 + 5 + 5$.

Rule for Multiplication.—1. *Write the multiplier under the multiplicand, placing units under units, tens under tens, etc.*

2. *When the multiplier consists of but one term, begin at the right and multiply successively each term of the multiplicand, writing the right-hand term of each result in the product and adding the left-hand term to the next result.*

3. *When the multiplier consists of more than one term, multiply the multiplicand successively by each significant*

term of the multiplier, writing the first term of each partial product under the term of the multiplier which produces it.

4. *Add the partial products thus obtained, and the sum will be the true product.*

ART. 34. 1. When the multiplier or multiplicand, or both, end with one or more ciphers:

Rule.—*Omit the ciphers in forming the partial products, and annex them to the sum of the partial products thus obtained.*

2. To multiply any number by 10, 100, etc.:

Rule.—*Annex to it as many ciphers as there are ciphers in the multiplier.*

LESSON XVII.

ADDITION, SUBTRACTION, AND MULTIPLICATION COMBINED.

ORAL PROBLEMS.

1. Multiply the sum of 7 and 13 by 5; by 8.

2. Multiply the difference of 13 and 7 by 8; by 12.

3. Multiply the sum of 13 and 7 by their difference.

4. Multiply the sum of 9 and 6 by their difference.

5. From the product of 6 and 4 take their sum.

6. From the square of 8 take the sum of 6 and 8.

7. From the square of 7 take the difference of 15 and 6.

8. From the sum of 27 and 9 take the square of 4.

9. A grocer bought 8 barrels of flour at $7 a barrel, and sold the lot for $60: how much did he gain?

10. A grocer bought 10 barrels of apples at $4 a barrel, and sold them at a gain of $15: for how much did he sell them?

11. A hatter paid $44 for a dozen silk hats, and sold them at $5 apiece: how much did he gain?

12. A lady teacher receives $9 a week, and spends $6 for board and washing: how much can she save in 8 weeks? In 12 weeks?

13. If a man earn $12 a week and spend $7, how much will he save in 9 weeks?

14. A drover bought 10 sheep at $6 a head, and 4 lambs at $3 a head, and then sold the lot for $80: how much did he gain?

15. If a man earn $8 a week, and a boy $3, how much will they both earn in 7 weeks?

WRITTEN PROBLEMS.

16. Multiply the sum of 940 and 560 by 240.

17. Multiply the difference of 940 and 560 by 520.

18. Multiply the sum of 940 and 560 by their difference.

19. From the product of 524 and 60 take their sum.

20. From the product of 608 and 75 take their difference.

21. A grocer bought 75 barrels of flour for $475, and sold it at $7 a barrel: what did he gain?

22. A clerk receives $125 a month, and spends $68 a month: how much does he lay up each year?

23. An agent sold 48 sets of maps at $16 a set; if the maps cost him $10 a set, how much did he make?

24. A man bought 100 acres of land for $4550, and then sold 60 acres of it at $56 an acre, and the remainder at $38 an acre: how much did he gain?

25. A man bought a farm for $4780, and sold 80 acres at $33 an acre, and the remaining portion for $2560: how much did he make by the transaction?

26. A drover bought 180 head of cattle in Ohio at $45 a head, shipped them to New York at an expense of $6 a head, and then sold them at $56 a head: how much did he make?

27. A miller manufactured 560 barrels of flour, and sold it at $9 a barrel; the wheat cost $2750, and the expense of running the mill was $960: how much did he make?

28. A man sold 5 horses at $87 apiece, and received $350 in cash and a note for the balance: what was the value of the note?

29. The President's salary is $50000 a year: if his expenses are $2500 a month, how much can he save during his term of 4 years?

30. A man earns $1800 a year and his expenses average $4 a day: how much will he save in 365 days, or one year? How much in 9 years?

DIVISION.

LESSON XVIII.

ORAL EXERCISES.

1. How many times is 2 contained in 6? 2 in 10? 2 in 14? 2 in 18? 2 in 16? 2 in 12?

2. How many times 3 in 9? 3 in 15? 3 in 21? 3 in 27? 3 in 12? 3 in 18? 3 in 24? 3 in 30? 3 in 36? 3 in 33? 3 in 32?

SUGGESTION.—3 is contained in 32 ten times, with 2 remainder.

3. How many times 4 in 8? 4 in 16? 4 in 24? 4 in 25? 4 in 26? 4 in 32? 4 in 40? 4 in 20? 4 in 28? 4 in 30? 4 in 36? 4 in 45?

4. How many times 5 in 25? 5 in 35? 5 in 45? 5 in 55? 5 in 30? 5 in 42? 5 in 52? 5 in 62?

5. How many times 6 in 18? 6 in 30? 6 in 42? 6 in 54? 6 in 56? 6 in 58? 6 in 72? 6 in 50? 6 in 38? 6 in 48? 6 in 52?

6. How many times 7 in 28? 7 in 42? 7 in 56? 7 in 70? 7 in 77? 7 in 84? 7 in 35? 7 in 53? 7 in 73? 7 in 75? 7 in 50? 7 in 53?

7. How many times 8 in 32? 8 in 40? 8 in 64? 8 in 56? 8 in 72? 8 in 88? 8 in 84? 8 in 90?

8. How many times 9 in 27? 9 in 45? 9 in 48? 9 in 63? 9 in 68? 9 in 72? 9 in 77? 9 in 95? 9 in 98? 9 in 108? 9 in 110?

9. How many times 10 in 50? 10 in 65? 10 in 96? 10 in 95? 10 in 98? 10 in 110? 10 in 120?

10. How many times 11 in 55? 11 in 77? 11 in 66? 11 in 88? 11 in 110? 11 in 99? 11 in 120?

11. How many times 12 in 36? 12 in 60? 12 in 84? 12 in 72? 12 in 96? 12 in 108? 12 in 121? 12 in 132?

12. How many 7's in 56? 8's in 72? 9's in 63? 6's in 54? 10's in 90? 12's in 72? 12's in 96? 11's in 99? 11's in 121?

WRITTEN EXERCISES.

13. Divide 80484 by 4.
14. Divide 930690 by 3.
15. Divide 808408 by 4.
16. Divide 120606 by 2; by 6.
17. Divide 84864 by 6.
18. Divide 18480 by 6; by 8.
19. Divide 87507 by 7; by 9.
20. Divide 648240 by 6; by 8.
21. Divide 398604 by 3; by 6.
22. Divide 10784 by 5; by 9.

PROCESS.

4)80484

20121, *Quotient.*

The above written process is called *short division.*

When there is a remainder after dividing the last term of the dividend, write the remainder below; or write the remainder over the divisor with a line between them and at the right of the quotient. 10784 divided by 5 gives 2156, with 4 as a remainder, or $2156\frac{4}{5}$. *The latter form is* preferable.

PROCESS.

5)10784

2156, *Quotient.*
4, *Remainder.*

9)10784

$1198\frac{2}{9}$, *Quotient.*

23. Divide 1606075 by 4; by 6; by 5; by 8.
24. Divide 4706270 by 3; by 4; by 7; by 9.
25. Divide 10632403 by 5; by 6; by 8; by 7.
26. Divide $207841 by 3; by 6; by 8; by 9.
27. Divide $4038475 by 5; by 4; by 6; by 8.
28. Divide 543020 by 5; by 10; by 11; by 12.
29. Divide 1030507 by 3; by 5; by 7; by 9.

LESSON XIX.

LONG DIVISION.

1. Divide 60703 by 7 by long division.
2. Divide 4876 by 23.

Divide 48 hundreds by 23, and write the result, 2 hundreds, at the right of the dividend for the first or hundreds' term of the quotient. Multiply the divisor (23) by this quotient term, and subtract the product, 46 hundreds, from 48 hundreds. To the remainder, 2 hundreds, annex the 7 tens of the dividend, giving 27 tens for the *second partial dividend.*

PROCESS.

23)4876(212, *Quotient.*
46
27
23
46
46

Divide 27 tens by 23, and write the result, 1 ten, for the tens' term of the quotient. Multiply 23 by this 1 ten, and subtract the product, 23 tens, from 27 tens. To the remainder, 4 tens, annex the 6 units of the dividend, giving 46 units for the *third partial dividend.*

ILLUSTRATIVE PROCESS.

23	4600	200
23	230	10
23	46	2

23)4876(212

Divide 46 units by 23, and write the result, 2, for the units' term of the quotient. Multiply 23 by 2, and subtract the result from 46. The quotient is 212.

NOTE.—In this and the next 14 problems, each term of the quotient may be determined *by dividing the left-hand term of the partial dividend by the left-hand term* of the divisor.

3. Divide 4664 by 22.
4. Divide 2825 by 25.
5. Divide 4686 by 22.
6. Divide 68952 by 221.
7. Divide 3813 by 123.
8. Divide 63336 by 203.
9. Divide 446886 by 2013.
10. 678273 by 2113.
11. 549661 by 1043.
12. 6989818 by 1067.
13. 4890375 by 1035.
14. 3585466 by 1043.
15. 4569121 by 1056.
16. 886784 by 2048.

17. 16080 by 67.

PROCESS.

67)16080(240, *Quotient.*
134
268
268
0

Since 67 is not contained in the number denoted by the first two left-hand terms of the dividend, take 160 *hundreds* for the *first partial dividend.*

The last partial dividend is 0, and the last quotient figure is 0.

18. Divide 312048 by 24.
19. Divide 374051 by 17.
20. Divide 79272 by 36.
21. Divide 1625130 by 65.
22. Divide 25272 by 36.
23. Divide 302526 by 63.
24. Divide 64347 by 267.
25. Divide 49179 by 507.
26. 54054 by 91.
27. 1200484 by 79.
28. 447125 by 73.
29. $21170 by 365.
30. $28431 by 351.
31. $212992 by 52.
32. 171814 by 634.
33. 120119 by 485.

34. Divide 34137 by 84.

PROCESS.

84)34137(406, *Quo.*
336
537
504
33, *Remainder.*

Since the divisor is not contained in the second partial dividend (53 tens), write 0 in the tens' place in the quotient, and annex the 7 units for a *third* partial dividend. As there is no figure of the dividend left to annex to 33 to form a new partial dividend, 33 remains undivided, and is called the *remainder.* The remainder may also be written over the divisor at the right of *the quotient, thus:* $406\frac{33}{84}$.

35. Divide 24399 by 48.
36. Divide 467034 by 806.
37. Divide 2845007 by 5728.
38. Divide 215607 by 1806.
39. Divide 1423685 by 6785.
40. 1604083 by 2088.
41. 85176 by 168.
42. 268272 by 608.
43. 569536 by 704.
44. 4859663 by 809.

45. Divide 350 by 10.

FIRST PROCESS.

10)350(35, *Quotient.*
30
50
50

SECOND PROCESS.

1|0)35|0
35, *Quotient.*

By comparing these two processes, it is seen that 350 is divided by 10 by cutting off the right-hand figure. The cutting off of the right-hand figure removes each of the other figures one place to the right, and thus divides their value by 10. Cutting off the two right-hand figures divides a number by 100; the three right-hand figures, by 1000, etc.

46. Divide 2800 by 100.
47. Divide 45600 by 10; by 100.
48. Divide 187000 by 1000; by 100.
49. Divide 384050 by 100; by 1000.
50. Divide 230045 by 1000; by 10000.
51. Divide 450860 by 10000; by 1000.

52. Divide 196800 by 4800.

PROCESS.

48|00)1968|00(41, *Quotient.*
192
48
48

First divide both divisor and dividend by 100, which is done by cutting off the two right-hand figures. Then divide 1968, the new dividend, by 48, the new divisor. The quotient is 41.

NOTE.—The teacher can show that *both divisor and dividend may be divided by any number without affecting the value of the quotient.*

53. Divide 63200 by 7900.
54. Divide 116087000 by 2900.
55. Divide 70125000 by 75000.

56. Divide 58864 by 4500.

PROCESS.

45|00)588|64(13, *Quotient.*
45
138
135
364, *Remainder.*

First divide both divisor and dividend by 100, which, in the case of the dividend, leaves a remainder of 64. Then divide 588 by 45, which leaves a remainder of 3 (*hundreds*), and to the 3 hundreds annex 64, the first remainder, thus obtaining 364 for the true remainder.

57. Divide 466384 by 3900.
58. Divide 99990 by 5400.
59. Divide 220345 by 940.
60. Divide 172800 by 14400.
61. Divide 219075 by 4500.
62. 39688000 by 4400.
63. 384500 by 4500.
64. 760724 by 7400.
65. 1752000 by 87600.
66. 126400 by 15800.

67. The divisor is $15000 and the dividend $5640000: what is the quotient?

68. The product of two numbers is 2364000, and one of the numbers is 2400: what is the other number?

LESSON XX.

ORAL PROBLEMS.

1. If a man walk 3 miles an hour, how long will it take him to walk 15 miles?

SOLUTION.—It will take as many hours as 3 miles are contained times in 15 miles, which is 5: it will take 5 hours.

2. *In an* orchard there are 16 trees, in rows of 4 trees *each: how many rows* in the orchard?

3. How many ranks of 4 soldiers each will 24 soldiers make? 32 soldiers? 40 soldiers?

4. A man planted 30 peach-trees in rows, setting 5 trees in each row: how many rows did they make?

5. Mary is reading 5 chapters a day: how long will it take her to read 45 chapters? 60 chapters?

6. A boy had 50 peach-stones, which he planted in rows of 5 each: how many rows did he plant?

7. If 6 chairs make a set, how many sets will 36 chairs make? 48 chairs? 60 chairs? 72 chairs?

8. There are 7 days in a week: how many weeks in 49 days? In 56 days? 63 days? 77 days?

9. There are 6 feet in a fathom: how many fathoms in 54 feet? In 60 feet? 72 feet? 66 feet?

10. There are 8 quarts in a peck: how many pecks in 72 quarts? 48 quarts? 64 quarts?

11. If a steamer run 8 miles an hour, in how many hours will it run 80 miles? 72 miles?

12. There are 8 furlongs in a mile: how many miles in 56 furlongs? 64 furlongs? 80 furlongs?

13. If a man work 8 hours a day, in how many days will he work 72 hours? 96 hours?

14. How long will it take a steamer to make a trip of 81 miles if it run 9 miles an hour?

15. If 9 words fill a line, how many lines will 72 words fill? 81 words? 90 words?

16. How many chairs, at $4 apiece, can be bought for $36? For $40? $48? $28? $32?

17. How many pairs of boots, at $5 a pair, can be bought for $35? For $55? $45? $25? $40?

18. How many plows, at $6 each, can be bought for $48? For $54? $60? $72? $36? $42?

19. How many sheep, at $9 a head, can be bought for $54? For $63? $90? $108? $81? $72?

20. How many tons of hay, at $10 a ton, can be bought for $90? For $120? $150? $160?

WRITTEN PROBLEMS.

21. At \$3 a bushel, how many bushels of wheat can be bought for \$963? For \$639?

22. In how many hours can a man walk 396 miles, if he walk at the rate of 3 miles an hour?

23. If a man earn \$2 a day, how long will it take him to earn \$360? \$350?

24. A manufacturer packed 372 clocks in boxes, with 4 clocks in each box: how many boxes were used?

25. If 4 bushels of wheat will make a barrel of flour, how many barrels will 972 bushels make?

26. If a man earn \$4 a day, how many days will it take him to earn \$1584? \$2140?

27. There are 36 inches in a yard: how many yards are there in 792 inches? 2376 inches?

28. A bushel of corn weighs 56 pounds: how many bushels of corn in 244160 pounds?

29. A hogshead of molasses contains 63 gallons: how many hogsheads in 47880 gallons?

30. If 72 books can be packed in a box, how many boxes will it take to hold 174960 books?

31. How many farms of 156 acres each can be sold from a tract of land containing 7332 acres?

32. If a vessel sail, on an average, 47 miles a day, how long will it take it to sail 2303 miles?

33. There are 365 days in a common year: how many years are there in 901550 days?

34. A pipe discharges 94 gallons in an hour: in how many hours will it empty a cistern holding 3384 gallons of water?

35. In 1 week there are 168 hours: how many weeks in 85008 hours? In 254016 hours?

36. A drover went West with \$23490 to buy cattle: *how many cattle* could he buy at \$58 a head?

37. If a garrison consume 648 pounds of bread in a day, how long will 134136 pounds last it?

38. If the average daily receipts of a ferry-boat be $275, in how many days will its receipts be $165825?

39. A barrel of beef contains 200 pounds: how many barrels will contain 128000 pounds?

40. There are 480 sheets of paper in a ream: how many reams will 1296000 sheets make?

41. There are 3600 seconds in an hour: how many hours in 17280000 seconds?

42. How many city lots, at $1600 each, can be bought for $25600? For $768000?

43. How many cars, each carrying 18000 pounds, will be required to transport 792000 pounds of hay?

44. How many regiments, averaging 750 men each, will make an army of 30000 men?

LESSON XXI.

DEFINITIONS, PRINCIPLES, AND RULES.

ART. 35. **Division** is the process of finding how many times one number is contained in another.

The **Dividend** is the number divided.

The **Divisor** is the number by which the dividend is divided.

The **Quotient** is the number of times the divisor is contained in the dividend.

The **Remainder** is the part of the dividend which is left undivided.

When the dividend contains the divisor an exact number of times, there is no remainder.

ART. 36. The **Sign of Division** is ÷, and is read *divided by.* When placed between two numbers, it

shows that the number before it is to be divided by the number after it. Thus: $16 \div 4 = 4$ is read 16 *divided by* 4 *equals* 4.

Division is also expressed by writing the dividend above and the divisor below a short horizontal line. $\frac{18}{13}$ may be read 18 *divided by* 13.

ART. 37. A number is contained in another as many times as it is taken to produce it. Hence, *division is the reverse of multiplication.*

The divisor and quotient are *factors* of the dividend.

One number is contained in another number as many times as it can be taken from it. Hence, division may be regarded as *a short method of subtraction.*

ART. 38. There are two methods of division, called *Short Division* and *Long Division.*

In *Short Division*, the partial products and partial dividends are not written, but are formed mentally.

In *Long Division*, the partial products and partial dividends, as well as the quotient, are written.

Rule for Short Division.—1. *Write the divisor at the left of the dividend, draw a curved line between them, and a straight line under the dividend.*

2. *Find how many times the divisor is contained in the left-hand term or terms of the dividend, taken as a partial dividend, and write the quotient under the last figure of the dividend used.*

3. *Multiply the divisor by the quotient term found, and subtract the product from the partial dividend used, performing each process mentally.*

4. *Prefix the remainder, if there be one, to the next term of the dividend for a second partial dividend, and divide, multiply, and subtract, as before.*

5. *Proceed in this manner until all the terms of the dividend have been used.*

Rule for Long Division.—1. *Write the divisor at the left of the dividend, draw a curved line between them, and also at the right of the dividend, to separate it from the quotient.*

2. *Take as many of the left-hand terms of the dividend as will contain the divisor for a partial dividend; find how many times this will contain the divisor, and write the quotient at the right of the dividend for the first left-hand term of the quotient.*

3. *Multiply the divisor by the quotient term found, write the product under the partial dividend used, and subtract.*

4. *To the remainder annex the next term of the dividend for a second partial dividend, and divide, multiply, and subtract, as before.*

5. *Proceed in this manner until all the terms of the dividend have been used.*

NOTE.—When any partial dividend does not contain the divisor, write a cipher in the quotient, and annex another term of the dividend to form a new partial dividend.

Proof.—*Multiply the divisor by the quotient, to the product add the remainder, if there be any, and if the result equals the dividend, the work is correct.*

ART. 39. When one or more of the right-hand figures of the divisor are ciphers:

Rule.—1. *Cut off the ciphers from the right of the divisor, and an equal number of figures from the right of the dividend.*

2. *Divide the new dividend thus formed by the new divisor, and the result will be the quotient.*

3. *Prefix the figures which denote the remainder, if there be one, to the figures cut off from the dividend, and the result will express the true remainder.*

ART. 40. To divide any number by 10, 100, etc.,—

Rule.—*Cut off as many figures from the right as there are ciphers in the divisor. The figures cut off express the remainder.*

FACTORS, DIVISORS, AND MULTIPLES.

LESSON XXII.

FACTORS.

NOTE.—The terms *number*, *factor*, and *divisor*, used in this and the next two lessons, denote *integers*.

ORAL EXERCISES.

1. What two numbers multiplied together will produce 6? 10? 14? 15? 21? 25? 27?

2. What two numbers multiplied together will produce 12? 16? 18? 20? 24? 30? 36? 40?

ART. 41. The numbers which, multiplied together, will produce a number, are called its *Factors*.

3. What are the factors of 15? 25? 35? 45? 55? 21? 27? 36? 42? 63?

4. What are the factors of 20? 28? 32? 48? 54? 56? 50? 63? 64? 72?

5. Give all the pairs of factors of 12.

ANSWER.—12 is 3 times 4 or 6 times 2.

6. Give all the pairs of factors of 16; 18; 20; 24; 30; 36; 40.

7. What three numbers multiplied continuously will produce 12?

ANSWER.—2, 2, and 3; $2 \times 2 \times 3 = 12$.

8. What three numbers multiplied continuously will produce 18? 20? 27? 28? 30?

9. What three numbers multiplied continuously will *produce 42? 45?* 50? 70? 60?

ART. 42. A number which is the product of two other numbers is a **Composite Number.** 6 is composite, being the product of 2 and 3.

10. Name all the composite numbers from 0 to 10; 10 to 20; 20 to 30; 30 to 40.

ART. 43. A number which is not the product of two other numbers is a **Prime Number.**

11. Name all the prime numbers from 0 to 10; 10 to 20; 20 to 30; 30 to 40.

12. Give all the prime factors of 12, not including 1.

ANSWER.—2, 2, and 3; $2 \times 2 \times 3 = 12$.

13. Give all the prime factors of 16; 18; 20; 24; 28; 30; 32; 36; 40.

14. Of what number are 2, 2, and 3 the prime factors? 2, 3, and 3? 2, 3, and 5? 2, 2, 3, and 5? 3, 5, and 7? 2, 3, 5, and 7? 2, 5, 7, and 11?

WRITTEN EXERCISES.

15. What are the prime factors of 60?

PROCESS.

$$\begin{array}{r} 2)60 \\ 2)30 \\ 3)15 \\ \hline 5 \end{array}$$

2, 2, 3, 5, *Pr. factors.*

What are the prime factors of;

16. 56? 63? 64? 72?

17. 84? 90? 96? 100?

18. 108? 140? 156? 216?

19. 81? 120? 144? 256?

20. 70? 124? 324? 280?

21. What prime factors are common to 54 and 72?

The common factors may be designated by drawing a line through them.

PROCESS.

$54 = 2 \times 3 \times 3 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

2, 3, 3, Common factors.

22. What prime factors are common to 42 and 54? 60 and 90? 84 and 108? 120 and 150? 70 and 140? 63 and 72?

23. What is the greatest factor common to 48 and 72?

Resolve 48 and 72 into their prime factors. The product of the common prime factors is the greatest common factor.

PROCESS.

$48 = 2 \times 2 \times 2 \times 2 \times 3$
$72 = 2 \times 2 \times 2 \times 3 \times 3$

2, 2, 2, 3, *Com. factors.*

$2 \times 2 \times 2 \times 3 = 24$, *G. C. F.*

What is the greatest common factor of;

24. 63 and 72?
25. 84 and 140?
26. 75 and 100?
27. 105 and 147?
28. 108 and 136?
29. 225 and 270?
30. 72 and 180?
31. 156 and 240?
32. 168 and 216?
33. 70, 140, and 175?
34. 108, 180, and 252?
35. 54, 108, and 162?
36. 75, 125, and 150?
37. 63, 84, 105?
38. 72, 96; 120?
39. 54, 90, 126?
40. 25, 100, 150?
41. 63, 108, 144?

ART. 44. A **Factor** of a number is one of the two or more numbers which, multiplied together, will produce it.

LESSON XXIII.

DIVISORS.

ORAL EXERCISES.

1. What numbers are divisors of 15? 18? 21? 28? 36? 42? 63? 72? 54?

2. What are the common divisors of 16 and 24? 15 and 45? 20 and 60? 35 and 70? 40 and 72?

3. What is the greatest common divisor of 15 and *45? 18 and 36?* 24 and 36? 50 and 75?

WRITTEN EXERCISES.

4. What is the greatest common divisor of 72 and 126?

Find the greatest common factor, and this will be the greatest common divisor.

PROCESS.

$72 = 2 \times 2 \times 2 \times 3 \times 3$
$126 = 2 \times 3 \times 3 \times 7$
$2 \times 3 \times 3 = 18$, *G. C. D.*

What is the greatest common divisor of;

5. 63 and 72?	**10.** 81, 135?	**15.** 54, 108, 162?
6. 72 and 96?	**11.** 108, 136?	**16.** 75, 125, 150?
7. 75 and 90?	**12.** 225, 270?	**17.** 63, 84, 105?
8. 72 and 180?	**13.** 312, 480?	**18.** 72, 96, 120?
9. 84 and 108?	**14.** 105, 147?	**19.** 108, 180, 252?

DEFINITIONS AND RULE.

ART. 45. The **Divisor** of a number is any number that will exactly divide it.

Every divisor of a number is a factor, and every factor is a divisor. The terms factor and divisor differ only in their origin, the former implying *multiplication* and the latter *division*. A divisor or factor of a number is also called a *measure*.

A **Common Divisor** of two or more numbers is a divisor of each of them.

The **Greatest Common Divisor** of two or more numbers is the greatest divisor of each of them.

A common divisor of two or more numbers is a common factor, and the greatest common divisor is the greatest common factor.

ART. 46. To find the greatest common divisor (G. C. D.) of two or more numbers:

Rule.—*Resolve the given numbers into their prime factors; select the factors which are common, and multiply them together. The product will be the* greatest common divisor.

LESSON XXIV.

MULTIPLES.

ORAL EXERCISES.

ART. 47. The product of two numbers is a *multiple* of each of them. 6 is a multiple of 2 and of 3; 10 is a multiple of 2 and of 5.

1. Of what numbers is 15 a multiple? 21? 35? 18? 24? 28? 36? 42? 50?

2. What number is a multiple of 3? 5? 7? 8?

3. What is a multiple of 12? 15? 20? 25? 30?

4. What is a common multiple of 3 and 4? 4 and 5? 5 and 10? 3 and 9? 7 and 10? 6 and 8? 4 and 12?

5. What is a common multiple of 2, 3, and 4? 3, 4, and 6? 4, 6, and 8? 3, 5, and 10? 2, 3, and 5?

6. What is the least common multiple of 3 and 4? 4 and 5? 6 and 8? 4 and 10? 5 and 6?

7. What is the least multiple common of 2, 3, and 4? 3, 4, and 6? 4, 6, and 8? 3, 5, and 10?

WRITTEN EXERCISES.

8. What is the least common multiple of 12, 18, and 30?

Resolve 12, 18, and 30 into their prime factors, and select all the *different* factors, repeating each as many times as it is found in any one of the numbers. The factor 2 occurs twice in 12; 3, twice in 18; and *5, once in 20.* The product of 2, 2, 3, 3, and 5 is the least *common multiple* of 12, 18, and 30.

PROCESS.

$12 = \not{2} \times \not{2} \times 3$
$18 = 2 \times \not{3} \times \not{3}$
$30 = 2 \times 3 \times \not{5}$

$2 \times 2 \times 3 \times 3 \times 5 = 180$, *L. C. M.*

What is the least common multiple of;

9. 12, 15, and 20?
10. 21, 24, and 42?
11. 32, 48, and 80?
12. 27, 54, and 108?
13. 24, 80, and 120?
14. 24, 18, and 48?
15. 12, 15, and 32?
16. 50, 75, and 300?
17. \$25, \$125, \$250?
18. \$48, \$72, \$36, \$144?
19. 18, 36, 54, 72?
20. 16, 32, 64, 128?
21. 15, 45, 90, 180?
22. 12, 24, 48, 96?
23. 25, 75, 150, 300?
24. 42, 84, 126, 252?

25. What is the least common multiple of 4, 6, 8, 12, 18, and 24?

26. What is the least common multiple of 3, 5, 6, 10, 15, 20, and 30?

27. What is the least common multiple of 2, 3, 4, 5, 6, 8, 10, and 15?

DEFINITIONS AND RULE.

ART. 48. A **Multiple** of a number is any number of which it is a factor.

A product is the multiple of each of its factors.

A **Common Multiple** of two or more numbers is a multiple of each of them.

The **Least Common Multiple** of two or more numbers is the least multiple of each of them.

ART. 49. To find the least common multiple (L. C. M.) of two or more numbers:

Rule.—*Resolve the given numbers into their prime factors; select all the different factors, taking each the highest number of times it is found in any one of the numbers, and multiply together the factors thus selected. The product will be the least common multiple.*

LESSON XXV.

REVIEW PROBLEMS.

1. The sum of two numbers is 15, and one of the numbers is 6: what is the other number?

SUGGESTION.—The other number is 15 less 6, which is 9.

2. The sum of two numbers is 19, and one of the numbers is 12: what is the other number?

3. The difference of two numbers is 8, and the smaller number is 9: what is the larger number?

4. The difference of two numbers is 7, and the smaller number is 8: what is the larger number?

5. The product of two numbers is 56, and one of the numbers is 7: what is the other number?

6. The product of two numbers is 72, and one of the numbers is 8: what is the other number?

7. The quotient is 6, and the divisor is 8: what is the dividend?

8. The quotient is 9, and the divisor is 12: what is the dividend?

9. At \$7 a barrel, how many barrels of flour will cost \$84? \$140? \$210?

10. If a vessel sails 72 miles in 9 hours, how many miles does it sail per hour?

SOLUTION.—If the vessel sails 72 miles in 9 hours, in 1 hour it sails one ninth of 72 miles, which is 8 miles. It sails 8 miles an hour.

11. If a man can dig 90 rods of ditch in 10 days, how many rods can he dig in 1 day?

12. If 9 barrels of flour cost \$63, how much does 1 barrel cost? 5 barrels? 6 barrels?

13. If 60 yards of cloth will make 12 suits of boys' *clothes,* how many yards will make 1 suit? How many *yards will make* 6 suits?

14. If a stage-coach runs 108 miles in 12 hours, how many miles does it run in 1 hour? In 5 hours?

15. Nine dictionaries cost $72; how much is that apiece?

16. How many barrels of flour at $8 a barrel will pay for 24 yards of carpeting at $2 a yard?

17. How many tons of coal at $9 a ton will pay for 15 cords of wood at $6 a cord?

18. A grocer bought 7 barrels of flour at $6 a barrel: for how much must he sell the lot to gain $14? For how much a barrel?

19. When oranges are sold at 5 cents apiece and lemons at 3 cents apiece, how many cents will buy 6 oranges and 8 lemons?

20. John has 6 marbles, and Willis has 4 times as many, and Charles has as many as both John and Willis: how many marbles has Charles?

21. Mary bought 6 yards of muslin at 8 cents a yard, 4 yards of braid at 3 cents a yard, and 3 spools of thread at 5 cents apiece, and gave the shop-keeper a silver dollar: how much change did she receive?

22. Two men start from the same place, and travel in opposite directions, one at the rate of 3 miles an hour and the other at 5 miles an hour: how far will they be apart in 1 hour? In 9 hours?

23. Two stages start from the same place and go in opposite directions, one at 7 miles an hour and the other at 5 miles an hour: how far will they be apart in 1 hour? In 20 hours?

24. Two stages start from the same place and go in the same direction, one at 9 miles an hour and the other at 6 miles an hour: how far will they be apart in 1 hour? In 10 hours?

25. Two vessels start from the same port and sail in the same direction, one sailing 15 miles an hour and the other 9 miles an hour: how far apart will they be in 1 hour? In 12 hours?

WRITTEN PROBLEMS.

26. The greater of two numbers is 4056, and their difference is 3650: what is the less number?

27. The subtrahend is 34203, and the remainder is 8706: what is the minuend?

28. The divisor is 534, and the quotient 43: what is the dividend?

29. What number multiplied by 98 will produce 15288?

30. The dividend is 5292, and the divisor is 63: what is the quotient?

31. The product of two numbers is 5328, and one of the numbers is 148: what is the other?

32. Multiply the sum of 486 and 392 by their difference.

33. Divide the product of 48 and 24 by their difference, and multiply the quotient by 50.

34. A merchant bought 35 yards of cloth for $56, and sold it at $2 a yard: how much did he gain?

35. A drover bought 240 sheep at $8 a head, and then sold 90 of them at $12 a head, 75 at $9 a head, and the rest at $6 a head: how much did he gain?

36. A farmer exchanged 65 bushels of wheat at $2 a bushel, and 35 sheep at $6 a head, for cows at $34 a head: how many cows did he receive?

37. A man's income is $3500 a year; he pays $450 a year for house-rent, $150 for taxes, $350 for hired help, and $45 a month for other expenses: how much are his expenses? How much has he left each year?

38. A man bought 80 acres of land at $35 an acre, paid $325 for improvements, and then sold it for $3750: how much did the land cost? How much did he gain?

39. A grain merchant, having 3500 bushels of oats, sold 1650 bushels, and then bought twice as much as *he had left: how* many bushels did he buy?

40. A widow has three sons and two daughters; the oldest son earns $30 a month, the second son $24 a month, the youngest son $10 a month, and each of the daughters $15 a month: how much do her five children earn in 1 month? How much in 12 months?

41. A grocer bought 350 bushels of potatoes at 60 cents a bushel, and paid 5 cents a bushel freight, and then sold the lot at 75 cents a bushel: how much did the potatoes cost him? How much did he gain?

42. A farmer sold 27 head of beef cattle at $46 a head, and 2 colts at $54 each, and then bought 35 young cattle at $18 a head, and 48 sheep at $6 a head: how much money had he left?

43. If a steamer carry, on an average, 75 passengers each trip, how many passengers will it carry in 12 weeks, making 3 trips a week?

44. A regiment contains 960 men, who receive, each, $16 a month, and the salary of all its officers is $2800 a month: what is the monthly pay of the regiment?

45. A man bought a farm containing 120 acres at $33 an acre, and another farm containing 80 acres at $45 an acre, and then sold both farms for $8750: how much did he gain?

46. A farmer raised 340 bushels of wheat, 525 bushels of corn, and 90 bushels of oats; he sold the wheat at 95 cents a bushel, the corn at 40 cents, and the oats at 35 cents: how much did he receive for his grain?

47. A man receives a salary of $1500 a year; he pays $20 a month for a house, $10 a month for help, $35 a month for meat and groceries, $250 a year for clothing, and $120 a year for other expenses: how much are his expenses each year? How much has he left?

48. A man starting on a journey took $200; he paid for railroad fare, $67; for berth in sleeper, 4 days, $2 a day; for hotel bills, 15 days, at $3 a day; and for other expenses, $25: how much money had he left?

QUESTIONS FOR REVIEW.

What is addition? What is meant by sum or amount? What does it contain? What is meant by like numbers? Give examples. What is the sign of addition? What does it show? Give the rule for addition. What is a method of proof?

What is subtraction? The difference? The minuend? The subtrahend? What kind of numbers can be subtracted? What does the sum of the remainder and the subtrahend equal? What is the sign of subtraction? What does it show? Give the rule for subtraction. What is the method of proof?

What is multiplication? The multiplicand? The multiplier? The product? What is the factor of a number? Of what are the multiplicand and multiplier factors?

What is the sign of multiplication? What does it show?

How may the product be obtained by addition?

Give the rule for multiplication. How may you multiply when either the multiplicand or multiplier, or both, end in ciphers? How may any number be multiplied by 10, 100, 1000, etc.?

What is division? The dividend? The divisor? The quotient? The remainder? Of what are the divisor and quotient factors?

What is the sign of division? What does it show? In what other way may division be expressed? Of what is division the reverse?

What is short division? When is it used? Give the rule. What is long division? Give the rule. What is a method of proof? How do you proceed when a partial dividend will not contain the divisor? How may you divide when the divisor ends in ciphers? How may any number be divided by 10, 100, etc.?

What is a factor of a number? What is a composite number? What is a prime number? What is a divisor of a number? What is a common divisor of two or more numbers? The greatest common divisor? Give the rule for finding the greatest common divisor of two or more numbers.

What is a multiple of a number? A common multiple of two or more numbers? The least common multiple? Give the rule for finding the least common multiple of two or more numbers.

PART III.

FRACTIONS.

LESSON I.

THE IDEA OF A FRACTION DEVELOPED.

1. If a melon be cut into two *equal* pieces, what part of the melon will one piece be?

2. How many halves in a melon? How many halves in any thing?

3. If a melon be cut into four equal pieces, what part of the melon will one piece be? Two pieces? Three pieces? Four pieces?

4. How many fourths in an apple? How many fourths in any thing?

5. Which is the greater, one half or one fourth of an apple? How many fourths equal one half?

6. If a cake be cut into three equal pieces, what part of the cake will one piece be? Two pieces?

7. How many thirds in a cake? How many thirds in any thing?

8. If a cake be cut into six equal pieces, what part of the cake will one piece be? Two pieces? Three pieces? Four pieces? Five pieces?

9. How many sixths in any thing?

10. Which is the greater, one third or one sixth of a cake? How many sixths equal one third?

11. A single thing is a *unit.* How many halves in a unit? How many thirds? How many fourths? How many fifths? How many sixths?

12. What is meant by one third?

Ans. One third is one of the three equal parts of a unit.

13. What is meant by two thirds? One fourth? Three fourths? One sixth? Three sixths?

14. What is meant by one fifth? Two fifths? Four fifths? Three fifths?

15. Which is the greater, two thirds or a unit? Five thirds or a unit? Three thirds or a unit?

16. Which is the greater, two fourths or a unit? Three fourths or a unit? Five fourths or a unit? Five sixths or a unit? Seven sixths or a unit?

ART. 50. Such parts of a unit as two thirds, three fourths, five sixths, etc., are called *Fractions.*

A fraction may be expressed by two numbers, one written under the other, with a horizontal line between them; as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

The number below the line is the number of equal parts into which the unit is divided. It is the *Denominator.*

The number above the line is the number of equal *parts taken.* It is the *Numerator.*

Read the following fractions, and in each case tell into how many equal parts the unit is divided, and how many parts are taken:

17.	18.	19.	20.	21.	22.
$\frac{3}{4}$	$\frac{6}{7}$	$\frac{5}{12}$	$\frac{5}{17}$	$\frac{6}{25}$	$\frac{20}{33}$
$\frac{5}{6}$	$\frac{9}{13}$	$\frac{8}{15}$	$\frac{13}{18}$	$\frac{18}{30}$	$\frac{25}{40}$
$\frac{7}{8}$	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{10}{16}$	$\frac{15}{20}$	$\frac{50}{50}$

Write the following fractions in figures:

23.	24.	25.
Two fifths.	Seven ninths.	Ten thirteenths.
Nine fifths.	Forty fiftieths.	Thirty-five fiftieths.
Ten ninths.	Eleven tenths.	Twenty-two twelfths.
Nine tenths.	Seven twelfths.	Twenty seventeenths.

DEFINITIONS.

ART. 51. A **Fraction** is one or more of the equal parts of a unit.

ART. 52. A fraction is expressed by two numbers, called the *Numerator* and the *Denominator*.

The **Denominator** of a fraction is the number of equal parts into which the unit is divided.

The **Numerator** of a fraction is the number of equal parts taken.

The numerator and denominator are called the *Terms* of a fraction.

ART. 53. A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{1}{8}$.

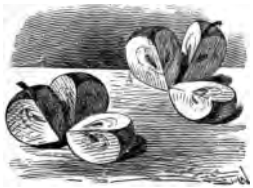
An **Improper Fraction** is one whose numerator is equal to or greater than the denominator; as, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{9}{7}$.

The value of a proper fraction is less than one; and the value of an improper fraction is equal to or greater than one.

LESSON II.

INTEGERS AND MIXED NUMBERS REDUCED TO FRACTIONS.

1. How many thirds in an apple? How many thirds in 2 apples?



2. How many fourths in an apple? How many fourths in 3 apples?

3. How many fourths in 2 pears? In 3 pears?

SOLUTION.—In 1 pear there are 4 fourths, and in 3 pears there are 3 times 4 fourths, which is 12 fourths. There are 12 fourths in 3 pears.

4. How many sixths in 3 oranges? In 5 oranges? 6 oranges? 4 oranges? 8 oranges?

5. How many fifths in 3? 5? 8? 10?

6. How many eighths in 4? 6? 8? 10?

7. How many halves in 2 and 1 half oranges?

SOLUTION.—In 2 oranges there are twice 2 halves, which is 4 halves, and 4 halves and 1 half are 5 halves. There are 5 halves in 2 and 1 half oranges.

8. How many fourths in 2? In 2 and 3 fourths?

9. How many thirds in 5? In 5 and 2 thirds?

10. How many thirds in 7 and 2 thirds? 6 and 2 thirds?

11. How many sixths in 5? In 5 and 2 sixths?

12. How many sixths in 10 and 5 sixths? 11 and 2 sixths?

13. How many tenths in $6\frac{3}{10}$? $5\frac{3}{10}$? $7\frac{4}{10}$? $8\frac{7}{10}$?

14. How many fifths in $6\frac{2}{5}$? $8\frac{3}{5}$? $12\frac{4}{5}$? $10\frac{3}{5}$? $9\frac{2}{5}$?

15. *How many* eighths in $5\frac{3}{8}$? $7\frac{4}{8}$? $9\frac{5}{8}$? $10\frac{7}{8}$? $12\frac{5}{8}$?

WRITTEN EXERCISES.

16. Reduce 157 to ninths; $157\frac{7}{9}$ to ninths.

PROCESS.

$1 = \frac{9}{9}$

$157 = \frac{1413}{9}$, *Ans.*

PROCESS.

$157\frac{7}{9}$

$\frac{1413}{7}$

$\frac{1420}{9}$, *Ans.*

17. Reduce $96\frac{5}{8}$ to eighths.

18. Reduce $46\frac{7}{12}$ to twelfths.

19. Reduce $63\frac{8}{11}$ to elevenths.

20. Reduce $53\frac{17}{20}$ to an improper fraction.

SUGGESTION.—Reduce the mixed number to *twentieths.*

Reduce the following mixed numbers to improper fractions:

21. $33\frac{4}{15}$.	**25.** $236\frac{4}{7}$.	**29.** $69\frac{27}{100}$.	**33.** $153\frac{7}{100}$.
22. $85\frac{9}{14}$.	**26.** $109\frac{7}{16}$.	**30.** $93\frac{9}{200}$.	**34.** $204\frac{15}{24}$.
23. $361\frac{1}{15}$.	**27.** $75\frac{9}{20}$.	**31.** $405\frac{5}{16}$.	**35.** $200\frac{13}{100}$.
24. $48\frac{5}{18}$.	**28.** $48\frac{17}{30}$.	**32.** $365\frac{1}{4}$.	**36.** $75\frac{41}{1000}$.

TO TEACHERS.—See "Manual of Arithmetic" for additional problems in this and the following lessons in Fractions.

ART. 54. A **Mixed Number** is an integer and a fraction united; as, $5\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{4}{7}$.

ART. 55. 1. To reduce an integer to a fraction:

Rule.—*Multiply the integer by the given denominator, and write the denominator under the product.*

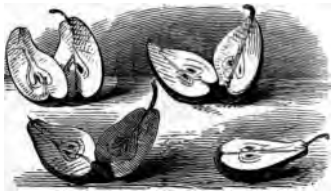
2. To reduce a mixed number to a fraction:

Rule.—*Multiply the integer by the denominator of the fraction, to the product add the numerator, and write the denominator under the result.*

LESSON III.

FRACTIONS REDUCED TO INTEGERS OR MIXED NUMBERS.

1. How many pears in 6 half-pears? In 7 half-pears?



2. How many pears in 10 half-pears? In 12 half-pears?

3. How many days in 11 half-days?

SOLUTION.—In 11 half-days there are as many days as 2 half-days are contained times in 11 half-days, which is $5\frac{1}{2}$ times. There are $5\frac{1}{2}$ days in 11 half-days.

4. How many pints in 14 half-pints? In 17 half-pints? In 21 half-pints?

5. How many yards in 18 thirds of a yard? In 19 thirds of a yard? In 22 thirds of a yard?

6. How many weeks in 28 sevenths of a week? 30 sevenths of a week?

7. A mason was 17 half-days in building a wall: how many days did he work?

8. How many ones in 25 fourths? In 30 fourths?

9. How many ones in 25 eighths? In 30 eighths?

10. How many ones in 35 tenths? In 45 tenths?

11. How many ones in 37 ninths? In 48 ninths?

12. How many ones in $\frac{45}{9}$? $\frac{56}{9}$? $\frac{75}{9}$? $\frac{84}{9}$? $\frac{90}{9}$?

13. How many ones in $\frac{60}{10}$? $\frac{65}{10}$? $\frac{77}{10}$? $\frac{98}{10}$? $\frac{100}{10}$?

WRITTEN EXERCISES.

14. Reduce $\frac{177}{8}$ to a mixed number.

PROCESS: $\frac{177}{8} = 177 \div 8 = 22\frac{1}{8}$, Ans.

PROCESS.

$8)177$

$22\frac{1}{8}$, Ans.

Reduce to an integer or mixed number:

15. $\frac{207}{12}$.	**20.** $\frac{75}{24}$.	**25.** $\frac{421}{21}$.	**30.** $\frac{630}{22}$.
16. $\frac{94}{15}$.	**21.** $\frac{312}{30}$.	**26.** $\frac{850}{85}$.	**31.** $\frac{795}{45}$.
17. $\frac{105}{12}$.	**22.** $\frac{160}{80}$.	**27.** $\frac{504}{41}$.	**32.** $\frac{908}{20}$.
18. $\frac{307}{20}$.	**23.** $\frac{220}{20}$.	**28.** $\frac{744}{85}$.	**33.** $\frac{1000}{86}$.
19. $\frac{860}{60}$.	**24.** $\frac{822}{16}$.	**29.** $\frac{815}{85}$.	**34.** $\frac{4005}{50}$.

ART. 56. To reduce an improper fraction to an integer or mixed number:

Rule.—*Divide the numerator of the fraction by the denominator.*

LESSON IV.

FRACTIONS REDUCED TO LOWER TERMS.

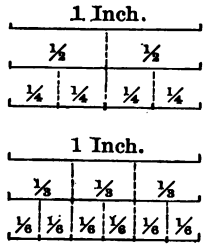
1. How many half-inches in 2 fourths of an inch? In 4 fourths of an inch?

2. How many halves in 2 fourths? In 4 fourths?

3. How many thirds of an inch in 2 sixths of an inch? In 4 sixths? In 6 sixths?

4. How many thirds in 2 sixths? In 4 sixths? In 6 sixths?

5. How many fourths in 4 eighths? In 6 eighths?



SOLUTION.—In 2 eighths there is 1 fourth, and in 6 eighths there are 3 fourths.

6. How many fifths in 2 tenths? In 4 tenths? 6 tenths? 8 tenths? 12 tenths? 14 tenths?

7. How many fifths in $\frac{6}{15}$? $\frac{9}{15}$? $\frac{12}{15}$?

NOTE.—The teacher should show that the value of a fraction is not changed by dividing both of its terms by the same number; and that $\frac{6}{15}$, $\frac{9}{15}$, and $\frac{12}{15}$ may each be changed to fifths by dividing both *terms by 3.*

8. How many sevenths in $\frac{6}{21}$? $\frac{12}{21}$? $\frac{15}{21}$?
9. How many eighths in $\frac{8}{32}$? $\frac{16}{32}$? $\frac{24}{32}$? $\frac{28}{32}$?
10. Reduce $\frac{9}{12}$, $\frac{8}{16}$, and $\frac{10}{20}$ each to fourths.
11. Reduce $\frac{18}{21}$, $\frac{25}{35}$, and $\frac{16}{28}$ each to sevenths.
12. Reduce $\frac{12}{24}$, $\frac{20}{32}$, $\frac{25}{40}$, and $\frac{18}{48}$ each to eighths.
13. Reduce $\frac{15}{30}$, $\frac{40}{80}$, $\frac{21}{42}$, and $\frac{24}{48}$ each to sixths.
14. Reduce $\frac{24}{40}$, $\frac{15}{60}$, $\frac{35}{70}$, and $\frac{36}{60}$ each to tenths.

WRITTEN EXERCISES.

15. Reduce $\frac{63}{84}$ to its lowest terms.

PROCESS.

$$\frac{63 \div 3}{84 \div 3} = \frac{21}{28}; \quad \frac{21 \div 7}{28 \div 7} = \frac{3}{4}$$

$$\text{Or: } \frac{63 \div 21}{84 \div 21} = \frac{3}{4}, \textit{ Ans.}$$

Reduce $\frac{63}{84}$ to $\frac{21}{28}$ by dividing both terms by 3; next reduce $\frac{21}{28}$ to $\frac{3}{4}$ by dividing both terms by 7; $\frac{3}{4}$ is in its *lowest terms*. Or, reduce $\frac{63}{84}$ to $\frac{3}{4}$ by dividing both terms by 21, the *greatest* number which will exactly divide each term.

Reduce to lowest terms:

16. $\frac{64}{72}$.
17. $\frac{72}{144}$.
18. $\frac{96}{144}$.
19. $\frac{56}{160}$.
20. $\frac{72}{108}$.
21. $\frac{32}{256}$.
22. $\frac{96}{100}$.
23. $\frac{84}{196}$.
24. $\frac{105}{175}$.
25. $\frac{13}{143}$.
26. $\frac{225}{315}$.
27. $\frac{182}{180}$.
28. $\frac{231}{360}$.
29. $\frac{144}{288}$.
30. $\frac{121}{242}$.
31. $\frac{360}{480}$.

ART. 57. When a fraction is reduced to an equivalent fraction with smaller terms, it is reduced to *lower terms.*

A fraction is in its *lowest terms* when no integer except 1 will exactly divide both numerator and denominator.

ART. 58. PRINCIPLE.—*The division of both terms of a fraction by the same number does not change its value.*

ART. 59. To reduce a fraction to its lowest terms:

Rule.—*Divide both terms of the fraction by any common divisor; then divide both terms of the resulting fraction by any common divisor; and so on, until the terms of the resulting fraction have no common divisor except 1.*

NOTE.—The reduction of a fraction to its lowest terms by finding the *greatest common divisor*, as in Art. 46, should be used *only* when a common divisor can not otherwise be found. All the fractions in this book may be easily reduced by using common prime divisors.

LESSON V.

FRACTIONS REDUCED TO HIGHER TERMS.

1. How many fourths of an orange in 1 half? In 2 halves?

2. How many eighths of an orange in 1 fourth of an orange? In 2 fourths?

3. How many eighths in 1 fourth? In 3 fourths?

SOLUTION.—1 fourth is 2 eighths, and 3 fourths are 3 times 2 eighths, which is 6 eighths.

4. How many ninths in 1 third? In 2 thirds? 3 thirds? 4 thirds?

5. How many tenths in $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{5}$? $\frac{6}{5}$?

6. How many twelfths in $\frac{2}{6}$? $\frac{4}{6}$? $\frac{5}{6}$? $\frac{3}{6}$? $\frac{7}{6}$?

7. Change $\frac{3}{4}$ and $\frac{5}{6}$ each to twelfths.

8. Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{9}$ each to eighteenths.

9. Change $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ each to twenty-fourths.

10. Change $\frac{2}{5}$, $\frac{7}{10}$, and $\frac{7}{15}$ each to thirtieths.

11. Change $\frac{3}{7}$, $\frac{11}{14}$, and $\frac{1}{2}$ each to twenty-eighths.

12. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{8}{12}$ to twenty-fourths.

13. Reduce $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{11}{20}$ to sixtieths.

WRITTEN EXERCISES.

14. Change $\frac{17}{35}$ to seventieths.

PROCESS.

$70 \div 35 = 2$

$\frac{17 \times 2}{35 \times 2} = \frac{34}{70}$, *Ans.*

One thirty-fifth is as many seventieths as 35 is contained times in 70, which is 2 times, and 17 thirty-fifths are 17 times 2 seventieths, which is 34 seventieths. This is the same as multiplying both terms by the quotient of 70 divided by 35.

15. Change $\frac{19}{24}$ to ninety-sixths.

16. Change $\frac{11}{24}$ and $\frac{23}{42}$ each to eighty-fourths.

17. Change $\frac{5}{12}$, $\frac{13}{24}$, and $\frac{15}{36}$ each to seventy-seconds.

18. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to equivalent fractions with a common denominator.

PROCESS: $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$; $\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$; $\frac{11 \times 2}{12 \times 2} = \frac{22}{24}$.

19. Reduce $\frac{7}{12}$, $\frac{5}{18}$, and $\frac{7}{9}$ to equivalent fractions with a common denominator.

20. Reduce $\frac{5}{7}$, $\frac{9}{14}$, and $\frac{16}{21}$ to equivalent fractions with a common denominator.

Reduce to equivalent fractions with a common denominator;

21. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$.

22. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{5}{6}$.

23. $\frac{3}{5}$, $\frac{7}{10}$, $\frac{8}{15}$.

24. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$.

25. $\frac{5}{1}$, $\frac{3}{7}$, $\frac{7}{14}$.

26. $\frac{1}{6}$, $\frac{5}{12}$, $\frac{7}{24}$.

27. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{4}{15}$, $\frac{11}{30}$.

28. $\frac{4}{5}$, $\frac{11}{12}$, $\frac{17}{18}$, $\frac{23}{36}$.

29. $\frac{9}{10}$, $\frac{13}{20}$, $\frac{14}{25}$, $\frac{33}{50}$.

ART. 60. When a fraction is changed to an equivalent fraction with greater terms, it is reduced to *higher terms.*

When several fractions have the same denominator, they have a *Common Denominator.*

ART. 61. PRINCIPLE.—*The multiplication of both terms of a fraction by the same number does not change its value.*

ART. 62. To reduce a fraction to higher terms:

Rule.—*Divide the given denominator by the denominator of the fraction, and multiply both terms by the quotient.*

ART. 63. To reduce fractions to equivalent fractions with a least common denominator:

Rule.—*Divide the least common multiple of the denominators by the denominator of each fraction, and multiply both of its terms by the quotient.*

LESSON VI.

ADDITION OF FRACTIONS.

1. A boy gave 1 fourth of a pine-apple to his brother, 1 fourth to his sister, and 1 fourth to a playmate: what part of it did he give away?

How many fourths are $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$?

2. A grocer sold 1 eighth of a cheese to one customer, 2 eighths to another, and 3 eighths to another: what part of it did he sell?

How much is $\frac{1}{8} + \frac{2}{8} + \frac{3}{8}$?

3. How many sixths in $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{5}{6}$? $\frac{3}{6}$, $\frac{4}{6}$, and $\frac{1}{6}$?

4. A boy gave 1 half of his money for a knife, and 1 third of it for a ball: what part of his money did he spend?

SUGGESTION.—Change $\frac{1}{2}$ and $\frac{1}{3}$ to sixths, and add the resulting fractions.

5. How many tenths in $\frac{1}{2}$ and $\frac{3}{5}$? $\frac{4}{5}$ and $\frac{3}{10}$?

6. How many twelfths in $\frac{1}{3}$ and $\frac{1}{4}$? $\frac{3}{4}$ and $\frac{5}{6}$?

7. How many eighths in $\frac{1}{4}$ and $\frac{1}{8}$? $\frac{3}{4}$ and $\frac{3}{8}$?

8. How many fifteenths in $\frac{1}{3}$ and $\frac{1}{5}$? $\frac{2}{3}$ and $\frac{2}{5}$?

9. How many twentieths in $\frac{1}{4}$ and $\frac{3}{5}$? $\frac{2}{5}$ and $\frac{3}{4}$? $\frac{3}{5}$ and $\frac{3}{4}$? $\frac{4}{5}$ and $\frac{3}{4}$? $\frac{1}{2}$ and $\frac{3}{10}$? $\frac{3}{4}$ and $\frac{3}{10}$?

10. How many twenty-fourths in $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$?

WRITTEN EXERCISES.

11. What is the sum of $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{9}{13}$?

PROCESS: $\frac{5}{13}+\frac{6}{13}+\frac{7}{13}+\frac{9}{13}=\frac{27}{13}=2\frac{1}{13}$, *Ans.*

12. What is the sum of $\frac{13}{24}$, $\frac{23}{24}$, $\frac{5}{24}$, and $\frac{19}{24}$?

13. What is the sum of $\frac{17}{30}$, $\frac{13}{30}$, $\frac{23}{30}$, and $\frac{23}{30}$?

14. What is the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$?

PROCESS.

$\frac{5}{6}+\frac{7}{8}+\frac{5}{12}=$

$\frac{20}{24}+\frac{21}{24}+\frac{10}{24}=\frac{51}{24}$

$\frac{51}{24}=2\frac{3}{24}=2\frac{1}{8}$, *Ans.*

Change the fractions to twenty-fourths; add the numerators of the new fractions and write the sum over the common denominator (24); and reduce the resulting improper fraction to a mixed number.

15. What is the sum of $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{15}$?

Add the following fractions:

16. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$.

17. $\frac{1}{6}$, $\frac{7}{12}$, $\frac{11}{24}$.

18. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{13}{15}$.

19. $\frac{1}{4}$, $\frac{3}{5}$, $\frac{17}{20}$.

20. $\frac{1}{5}$, $\frac{2}{3}$, $\frac{3}{15}$.

21. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$.

22. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$.

23. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$.

24. $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{30}$.

25. $\frac{2}{3}$, $\frac{5}{14}$, $\frac{9}{21}$, $\frac{11}{42}$.

26. $\frac{2}{5}$, $\frac{3}{4}$, $\frac{3}{10}$, $\frac{13}{20}$.

27. $\frac{5}{6}$, $\frac{5}{8}$, $\frac{5}{12}$, $\frac{17}{24}$.

28. $\frac{3}{7}$, $\frac{9}{14}$, $\frac{20}{21}$, $\frac{31}{42}$.

29. $\frac{9}{10}$, $\frac{11}{15}$, $\frac{3}{20}$, $\frac{23}{30}$.

30. What is the sum of $16\frac{2}{3}$, $18\frac{3}{4}$, and $37\frac{1}{2}$?

PROCESS.

$16\frac{2}{3}$ $\frac{8}{12}$

$18\frac{3}{4}$ $\frac{9}{12}$

$37\frac{1}{2}$ $\frac{6}{12}$

$72\frac{11}{12}$, *Ans.*

First add the fractions and then the integers. $\frac{2}{3}=\frac{8}{12}$, $\frac{3}{4}=\frac{9}{12}$, $\frac{1}{2}=\frac{6}{12}$. $\frac{8}{12}+\frac{9}{12}+\frac{6}{12}=\frac{23}{12}=1\frac{11}{12}$. Write the $\frac{11}{12}$ under the fractions and add the 1 with the integers.

31. *What is the* sum of $365\frac{1}{4}$, $408\frac{5}{6}$, and 340.

Add the following mixed numbers:

32. $45\frac{1}{2}$, $67\frac{3}{4}$, $62\frac{5}{8}$.
33. $37\frac{2}{3}$, $18\frac{3}{4}$, $33\frac{1}{3}$, $25\frac{7}{12}$.
34. $30\frac{1}{4}$, $66\frac{2}{3}$, $84\frac{3}{4}$, $133\frac{1}{3}$.
35. $75\frac{1}{2}$, 108, $160\frac{3}{5}$, 207.
36. $166\frac{2}{3}$, 409, $3047\frac{1}{2}$.
37. $102\frac{4}{7}$, $156\frac{7}{21}$, $905\frac{1}{3}$.
38. $30\frac{1}{4}$, $67\frac{1}{2}$, $3430\frac{3}{8}$.
39. 440, $90\frac{15}{42}$, $650\frac{7}{12}$.

ART. 64. 1. To add fractions:

Rule.—*Reduce the fractions to equivalent fractions with a common denominator, add the numerators of the new fractions, and under the sum write the common denominator.*

2. To add mixed numbers:

Rule.—*Add the fractions and the integers separately, and combine the results.*

LESSON VII.

SUBTRACTION OF FRACTIONS.

1. Albert had 2 thirds of an orange, and he gave 1 third to his sister: how many thirds had he left?

How much is $\frac{2}{3}$ less $\frac{1}{3}$? $\frac{3}{3}$ less $\frac{2}{3}$?

2. Charles bought 3 fourths of a pound of raisins, and then gave 1 fourth of a pound to his playmate: what part of a pound had he left?

How much is $\frac{3}{4}$ less $\frac{1}{4}$? $\frac{3}{4}$ less $\frac{2}{4}$?

3. A farmer bought $\frac{2}{3}$ of a bushel of flax-seed, and sold $\frac{1}{2}$ of a bushel to a neighbor: what part of a bushel had he left?

SUGGESTION.—Change $\frac{2}{3}$ and $\frac{1}{2}$ to sixths.

4. How much is $\frac{3}{4}$ less $\frac{1}{2}$? $\frac{3}{4}$ less $\frac{1}{8}$? $\frac{2}{8}$ less $\frac{1}{2}$?
5. How much is $\frac{7}{8}$ less $\frac{3}{4}$? $\frac{7}{8}$ less $\frac{2}{8}$? $\frac{7}{8}$ less $\frac{1}{4}$?
6. How much is $\frac{7}{12}$ less $\frac{1}{3}$? $\frac{7}{12}$ less $\frac{1}{2}$? $\frac{7}{12}$ less $\frac{1}{4}$?
7. How much is $\frac{9}{10}$ less $\frac{3}{4}$? $\frac{9}{10}$ less $\frac{4}{5}$? $\frac{9}{10}$ less $\frac{1}{2}$?
8. How much is $\frac{15}{16}$ less $\frac{5}{8}$? $\frac{15}{16}$ less $\frac{3}{4}$? $\frac{15}{16}$ less $\frac{7}{8}$?
9. How much is $\frac{7}{9}$ less $\frac{1}{6}$? $\frac{8}{15}$ less $\frac{3}{10}$? $\frac{3}{4}$ less $\frac{3}{8}$?

WRITTEN EXERCISES.

10. From $\frac{11}{24}$ take $\frac{7}{24}$.

PROCESS: $\frac{11}{24} - \frac{7}{24} = \frac{4}{24} = \frac{1}{6}$, *Ans.*

11. From $\frac{29}{54}$ take $\frac{11}{54}$.

12. From $\frac{67}{120}$ take $\frac{43}{120}$.

13. From $\frac{11}{12}$ take $\frac{5}{8}$.

PROCESS: $\frac{11}{12} - \frac{5}{8} = \frac{22}{24} - \frac{15}{24} = \frac{7}{24}$, *Ans.*

14. Subtract $\frac{3}{8}$ from $\frac{7}{12}$.

15. Subtract $\frac{7}{15}$ from $\frac{9}{10}$.

16. Subtract $\frac{12}{25}$ from $\frac{23}{50}$.

17. Subtract $\frac{5}{18}$ from $\frac{7}{12}$.

18. Subtract $\frac{5}{14}$ from $\frac{8}{21}$.

19. Subtract $\frac{5}{12}$ from $\frac{7}{15}$.

20. From $\frac{3}{5}$ take $\frac{5}{9}$.

21. From $\frac{13}{15}$ take $\frac{7}{10}$.

22. From $\frac{23}{25}$ take $\frac{13}{50}$.

23. From $\frac{7}{8}$ take $\frac{7}{12}$.

24. From $\frac{13}{14}$ take $\frac{11}{21}$.

25. From $\frac{11}{12}$ take $\frac{11}{18}$.

26. From $33\frac{1}{8}$ take $18\frac{3}{4}$.

PROCESS.

$33\frac{1}{8}$ $\quad \frac{4}{12}$

$18\frac{3}{4}$ $\quad \frac{9}{12}$

$14\frac{7}{12}$, *Ans.*

First subtract the fractions and then the integers. Since $\frac{9}{12}$ is greater than $\frac{4}{12}$, add $\frac{12}{12}$ to $\frac{4}{12}$, making $\frac{16}{12}$, and then take the $\frac{9}{12}$ from $\frac{16}{12}$, writing $\frac{7}{12}$ under the fractions, and adding 1 to the 8 units before subtracting the integers.

27. Subtract $30\frac{1}{4}$ from $66\frac{2}{3}$.

28. Subtract $112\frac{3}{4}$ from $145\frac{1}{2}$.

29. Subtract $250\frac{3}{8}$ from 300.

30. Subtract $130\frac{5}{8}$ from $241\frac{2}{3}$.

31. $45\frac{3}{5}$ from $66\frac{2}{3}$.

32. $90\frac{1}{3}$ from $108\frac{1}{2}$.

33. $105\frac{3}{4}$ from $261\frac{1}{3}$.

34. $166\frac{2}{3}$ from $233\frac{1}{3}$.

ART. 65. 1. To subtract fractions:

Rule.—*Reduce the fractions to equivalent fractions with a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend, and under the difference write the common denominator.*

2. To subtract mixed numbers:

Rule.—*First subtract the fractions, and then the integers.*

LESSON VIII.

PROBLEMS COMBINING ADDITION AND SUBTRACTION.

NOTE—The first eight of these problems are to be solved both orally and on the slate.

1. From the sum of $\frac{1}{2}$ and $\frac{1}{8}$ take $\frac{3}{4}$.
2. From the sum of $\frac{1}{4}$ and $\frac{4}{5}$ take $\frac{1}{2}$.
3. From $\frac{7}{8}$ take the sum of $\frac{1}{2}$ and $\frac{1}{4}$.
4. From $1\frac{1}{4}$ take the sum of $\frac{2}{8}$ and $\frac{1}{4}$.
5. From the sum of $\frac{2}{8}$ and $\frac{3}{4}$ take $1\frac{1}{4}$.
6. From $\frac{5}{8} + \frac{3}{4}$ take $\frac{1}{2} + \frac{1}{4}$.
7. From $\frac{7}{8} - \frac{1}{8}$ take $\frac{5}{8} - \frac{1}{2}$.
8. From 1 take $\frac{2}{5} + \frac{3}{10} + \frac{1}{4}$.
9. From 2 take $\frac{2}{3} + \frac{3}{4} + \frac{1}{6}$.
10. From the sum of $\frac{3}{4}$ and $\frac{2}{3}$ take their difference.
11. From $\frac{2}{3} + \frac{1}{4} + \frac{5}{6}$ take $\frac{7}{12}$.
12. From $\frac{3}{4} + \frac{5}{8} + \frac{2}{3}$ take $\frac{5}{12} - \frac{1}{6}$.
13. From $\frac{1}{3} + \frac{3}{4} + \frac{2}{5}$ take $\frac{1}{8} + \frac{1}{4} + \frac{1}{5}$.
14. From 1 take $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$.
15. From $3\frac{1}{3}$ take $\frac{2}{5} + \frac{7}{8} + \frac{5}{6} + \frac{2}{3}$.
16. To $\frac{5}{7} - \frac{2}{7}$ add $\frac{2}{7} + \frac{4}{5} + \frac{7}{10} + \frac{1}{4}$.
17. To $1\frac{1}{3} + \frac{5}{12}$ add $7\frac{1}{4} - \frac{1}{3}$.
18. To $2\frac{1}{5} - \frac{3}{4}$ add $3\frac{1}{3} - \frac{2}{5}$.
19. From $\$47\frac{3}{5} + \$33\frac{1}{3}$ take $\$47\frac{3}{5} - \$33\frac{1}{3}$.
20. From $\$256\frac{2}{3}$ take $\$140\frac{1}{3} + \$66\frac{1}{3}$.
21. To $150\frac{2}{3} - 94\frac{2}{5}$ add $106\frac{4}{5} - 71\frac{1}{3}$.
22. To $56\frac{1}{5} + 207\frac{3}{10}$ add $165\frac{1}{2} - 83\frac{3}{4}$.
23. From $16\frac{2}{3} + 12\frac{1}{2} + 5\frac{1}{4}$ take $18\frac{3}{4} + 6\frac{1}{3}$.
24. From $30\frac{1}{4} + 37\frac{1}{2}$ take $7\frac{5}{8} + 6\frac{5}{12} + 1\frac{3}{4}$.
25. To $66\frac{2}{3} - 30\frac{1}{2}$ add $77\frac{5}{12} - 33\frac{5}{8}$.
26. To $12\frac{3}{4} - 6\frac{1}{12}$ add $40 - 23\frac{3}{8}$.
27. The sum of two fractions is $\frac{3}{5}\frac{3}{0}$, and one of the fractions is $\frac{3}{10}$: what is the other?
28. The sum of two mixed numbers is $45\frac{3}{5}$, and one of the mixed numbers is $25\frac{5}{6}$: what is the other?

LESSON IX.

FRACTIONAL PARTS OF INTEGERS.

1. If 6 pears be divided equally between 2 boys, what part of the whole will each receive?

What is $\frac{1}{2}$ of 6 pears? $\frac{1}{2}$ of 10 pears?

2. A mother divided 5 apples equally between 2 children: what part of the whole did each receive?

What is $\frac{1}{2}$ of 5 apples?

SUGGESTION.—Take 1 half of 4 apples and 1 half of 1 apple.

3. Charles divided 12 plums equally between 3 boys: what part of the whole did each receive? How many plums did each receive?

4. What is $\frac{1}{3}$ of 9? $\frac{1}{3}$ of 12? $\frac{1}{3}$ of 16?

5. What is $\frac{1}{4}$ of 20? $\frac{1}{4}$ of 28? $\frac{1}{4}$ of 30?

6. What is $\frac{1}{5}$ of 25? $\frac{1}{5}$ of 26? $\frac{1}{5}$ of 37?

7. What is $\frac{1}{6}$ of 24? $\frac{5}{6}$ of 24?

SOLUTION.—$\frac{1}{6}$ of 24 is 4, and $\frac{5}{6}$ of 24 is 5 times 4, which is 20.

8. What is $\frac{1}{10}$ of 40? $\frac{5}{10}$ of 40? $\frac{7}{10}$ of 40?

9. What is $\frac{1}{7}$ of 63? $\frac{4}{7}$ of 63? $\frac{6}{7}$ of 63? $\frac{5}{7}$ of 63?

10. What is $\frac{2}{3}$ of 18? $\frac{3}{4}$ of 16? $\frac{4}{5}$ of 15? $\frac{5}{6}$ of 30?

11. What is $\frac{1}{3}$ of 24? 8 is $\frac{1}{3}$ of what number?

12. What is $\frac{1}{6}$ of 42? 7 is $\frac{1}{6}$ of what number?

13. 10 is $\frac{1}{6}$ of what number?

14. 12 is $\frac{1}{5}$ of what number?

15. What is $\frac{1}{10}$ of 60? $\frac{3}{10}$ of 60? $\frac{7}{10}$ of 60?

16. What is $\frac{1}{8}$ of 64? $\frac{3}{8}$ of 64? $\frac{5}{8}$ of 64?

WRITTEN EXERCISES.

17. What is $\frac{3}{8}$ of 659?
18. What is $\frac{2}{6}$ of 191?
19. What is $\frac{7}{8}$ of 508?
20. What is $\frac{6}{7}$ of 466?
21. What is $\frac{9}{22}$ of 906?
22. What is $\frac{6}{18}$ of 403?
23. What is $\frac{5}{12}$ of 348?
24. What is $\frac{5}{11}$ of 4642?
25. What is $\frac{18}{40}$ of 6080?

PROCESS.

$$\begin{array}{r} 8)659 \\ \hline 82\frac{3}{8} \\ 3 \\ \hline 247\frac{3}{8}, \textit{ Ans.} \end{array}$$

Or:

$$\begin{array}{r} 659 \\ \hline 3 \\ \hline 8)1977 \\ \hline 247\frac{1}{8} \end{array}$$

ART. 66. To find the fractional part of an integer:

Rules.—*Divide the integer by the denominator of the fraction and multiply the quotient by the numerator.* Or:

Multiply the integer by the numerator and divide the product by the denominator.

LESSON X.

FRACTIONAL PARTS OF FRACTIONS.

1. How much is $\frac{1}{2}$ of $\frac{1}{4}$ of a melon?

2. If a third of a melon be cut into two equal pieces, what part of the melon will each piece be?

3. What is $\frac{1}{2}$ of $\frac{1}{3}$ of a melon?

4. What is $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{2}$?

5. What is $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{7}$?

6. What is $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{7}$? $\frac{1}{5}$ of $\frac{1}{9}$? $\frac{1}{5}$ of $\frac{1}{6}$?

7. A girl, having $\frac{3}{4}$ of an orange, divided it equally between her 2 brothers: what part of the orange did each receive?

SUGGESTION.—Divide each fourth into 2 equal pieces, and then give 3 pieces to each. $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$, and $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

8. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{3}{4}$? $\frac{1}{2}$ of $\frac{5}{4}$? $\frac{1}{2}$ of $\frac{7}{4}$?
9. What is $\frac{1}{3}$ of $\frac{1}{8}$? $\frac{1}{3}$ of $\frac{3}{8}$? $\frac{1}{3}$ of $\frac{5}{8}$? $\frac{1}{3}$ of $\frac{7}{8}$?
10. What is $\frac{1}{5}$ of $\frac{5}{6}$? $\frac{1}{4}$ of $\frac{3}{7}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{3}{4}$ of $\frac{3}{5}$?

SOLUTION.—$\frac{1}{4}$ of $\frac{3}{5}$ is $\frac{3}{20}$, and $\frac{3}{4}$ of $\frac{3}{5}$ is 3 times $\frac{3}{20}$, which is $\frac{9}{20}$.

11. What is $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{3}{8}$ of $\frac{2}{7}$? $\frac{3}{5}$ of $\frac{5}{6}$?
12. What is $\frac{3}{7}$ of $\frac{7}{9}$? $\frac{3}{4}$ of $\frac{8}{7}$? $\frac{5}{6}$ of $\frac{7}{12}$? $\frac{2}{3}$ of $2\frac{1}{2}$?

WRITTEN EXERCISES.

13. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ to a simple fraction.

PROCESS: $\frac{2}{3}$ of $\frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$, *Ans.*

NOTE.—Since $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{3} \times \frac{3}{5}$, "of" between two fractions is equivalent to "$\times$".

Reduce to a simple fraction:

14. $\frac{3}{5}$ of $\frac{5}{7}$.
15. $\frac{5}{6}$ of $\frac{3}{5}$.
16. $\frac{6}{7}$ of $\frac{7}{12}$.
17. $\frac{3}{8}$ of $\frac{7}{12}$.
18. $\frac{5}{9}$ of $\frac{9}{11}$.
19. $\frac{6}{7}$ of $\frac{14}{15}$.
20. $\frac{4}{11}$ of $\frac{5}{12}$.
21. $\frac{7}{12}$ of $\frac{12}{21}$.
22. $\frac{3}{4}$ of $\frac{4}{5}$.
23. $\frac{3}{4}$ of $2\frac{1}{2}$.
24. $\frac{3}{7}$ of $2\frac{1}{3}$.
25. $\frac{4}{5}$ of $\frac{6}{7}$ of $1\frac{2}{3}$.
26. $\frac{3}{5}$ of $\frac{15}{16}$ of $5\frac{1}{3}$.
27. $\frac{4}{7}$ of $\frac{14}{15}$ of $1\frac{7}{8}$.
28. $\frac{5}{6}$ of $\frac{12}{18}$ of $1\frac{3}{10}$.
29. $\frac{3}{8}$ of $2\frac{2}{3}$ of $5\frac{1}{2}$.
30. $\frac{6}{7}$ of $\frac{14}{18}$ of $1\frac{1}{4}$.
31. $1\frac{7}{11}$ of $5\frac{1}{9}$ of $\frac{1}{46}$.

ART. 67. A **Simple Fraction** is a fraction not united with an integer or another fraction; as, $\frac{3}{4}$.

A **Compound Fraction** is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{5}$ of $3\frac{1}{3}$.

ART. 68. To reduce a compound fraction to a simple fraction:

Rule.—*Multiply the numerators together, and also the denominators, and reduce the resulting fraction to its lowest terms.*

NOTE.—The process may often be shortened by canceling common factors before multiplying. This process may be taught *after* the pupil is familiar with the other process.

LESSON XI.

FRACTIONS MULTIPLIED BY INTEGERS.

1. What part of a cake is twice 2 eighths of it? 3 times 2 eighths of it?

2. A father gave 3 fourths of an orange to each of 4 children: how many fourths did they all receive?

3. How much is 4 times $\frac{3}{4}$? 6 times $\frac{3}{4}$?

4. If a boy earn 2 thirds of a dollar in a day, how much will he earn in 3 days?

5. How much is 3 times $\frac{2}{8}$? 5 times $\frac{2}{8}$? 6 times $\frac{2}{8}$?

6. How much is 6 times $\frac{3}{5}$? 8 times $\frac{2}{5}$? 9 times $\frac{3}{5}$?

7. How much is 7 times $\frac{5}{6}$? 6 times $\frac{7}{8}$? 8 times $\frac{5}{9}$?

8. How much is 5 times $6\frac{2}{3}$? 7 times $8\frac{1}{2}$?

SUGGESTION.—Multiply the fraction and the integer separately, and add the products.

9. How much is 3 times $6\frac{2}{3}$? 3 times $7\frac{3}{4}$?

10. How much is 6 times $4\frac{5}{6}$? 6 times $8\frac{3}{7}$?

WRITTEN EXERCISES.

Multiply:

11. $\frac{5}{8}$ by 10.
12. $\frac{5}{12}$ by 8.
13. $\frac{7}{10}$ by 12.
14. $\frac{18}{20}$ by 24.
15. $\frac{21}{25}$ by 25.
16. $\frac{11}{45}$ by 16.
17. $\frac{13}{22}$ by 33.
18. $\frac{15}{32}$ by 16.
19. $16\frac{2}{3}$ by 8.
20. $18\frac{3}{4}$ by 12.
21. $66\frac{2}{3}$ by 34.
22. $83\frac{2}{5}$ by 25.

ART. 69. PRINCIPLE.—*A fraction is multiplied by multiplying its numerator or dividing its denominator.*

ART. 70. 1. To multiply a fraction by an integer:

Rule.—*Multiply the numerator or divide the denominator.*

2. To multiply a mixed number by an integer:

Rule.—*Multiply the fraction and the integer separately, and add the products.*

LESSON XII.

INTEGERS MULTIPLIED BY FRACTIONS.

1. How much is $\frac{1}{4}$ of 8? $\frac{3}{4}$ of 8? $\frac{3}{4}$ times 8?

SOLUTION.—$\frac{3}{4}$ times 8 is 3 times $\frac{1}{4}$ of 8; $\frac{1}{4}$ of 8 is 2, and 3 times 2 is 6. Or, $\frac{3}{4}$ times 8 is $\frac{3}{4}$ of 8, which is 6.

2. How much is $\frac{1}{3}$ of 12? $\frac{2}{3}$ of 12? $\frac{2}{3}$ times 12? $\frac{2}{3}$ times 30? $\frac{2}{3}$ times 21? $\frac{2}{3}$ times 24?

3. How much is $\frac{1}{6}$ of 18? $\frac{5}{6}$ of 18? $\frac{5}{6}$ times 18? $\frac{5}{6}$ times 24? $\frac{5}{6}$ times 36? $\frac{5}{6}$ times 42?

4. How much is $\frac{1}{5}$ of 15? $\frac{3}{5}$ of 15? $\frac{3}{5}$ times 15? $\frac{3}{5}$ times 35? $\frac{3}{5}$ times 45? $\frac{3}{5}$ times 50?

5. Multiply 40 by $\frac{2}{5}$; 40 by $\frac{5}{8}$; 40 by $\frac{3}{4}$.

6. Multiply 56 by $\frac{3}{7}$; by $\frac{5}{8}$; by $\frac{6}{7}$; by $\frac{7}{8}$.

WRITTEN EXERCISES.

7. Multiply 256 by $\frac{3}{4}$.

PROCESS.

$$\begin{array}{r} 4)256 \\ 64 \\ \underline{3} \\ 192, \textit{Ans.} \end{array} \qquad \text{Or:} \qquad \begin{array}{r} 256 \\ \underline{3} \\ 4)768 \\ 192 \end{array}$$

SOLUTION.—$\frac{3}{4}$ times 256 is 3 times $\frac{1}{4}$ of 256. Or, $\frac{3}{4}$ times 256 is $\frac{1}{4}$ of 3 times 256.

Multiply:

8. 48 by $\frac{5}{12}$.	**12.** 163 by $\frac{8}{11}$.	**16.** 248 by $\frac{12}{25}$.
9. 65 by $\frac{7}{8}$.	**13.** 300 by $\frac{13}{15}$.	**17.** 406 by $\frac{5}{21}$.
10. 59 by $\frac{6}{7}$.	**14.** 257 by $\frac{11}{16}$.	**18.** 856 by $\frac{13}{45}$.
11. 87 by $\frac{4}{5}$.	**15.** 305 by $\frac{21}{25}$.	**19.** 794 by $\frac{23}{36}$.

20. Multiply 324 by $6\frac{2}{3}$.

PROCESS.

324		324
$6\frac{2}{3}$	Or:	$6\frac{2}{3}$
1944		216
216		1944
2160, *Ans.*		2160, *Ans.*

First multiply by the integer and then by the fraction, and add the products; or, first multiply by the fraction and then by the integer.

Multiply:

21. 48 by $16\frac{1}{2}$.	**24.** 246 by $12\frac{3}{4}$.	**27.** 108 by $56\frac{3}{7}$.
22. 72 by $18\frac{3}{4}$.	**25.** 324 by $17\frac{1}{2}$.	**28.** 524 by $72\frac{3}{5}$.
23. 96 by $23\frac{7}{8}$.	**26.** 406 by $33\frac{1}{3}$.	**29.** 684 by $66\frac{2}{3}$.

ART. 71. 1. To multiply an integer by a fraction:

Rule.—*Divide the integer by the denominator and multiply the quotient by the numerator.* Or:

Multiply the integer by the numerator and divide the product by the denominator.

2. To multiply an integer by a mixed number:

Rule.—*Multiply by the integer and by the fraction separately, and add the products.*

LESSON XIII.

FRACTIONS MULTIPLIED BY FRACTIONS.

1. How much is $\frac{1}{4}$ times $\frac{2}{3}$? $\frac{3}{4}$ times $\frac{2}{3}$?

SOLUTION.—$\frac{1}{4}$ times $\frac{2}{3}$ is $\frac{1}{4}$ of $\frac{2}{3}$, which is $\frac{2}{12}$. $\frac{3}{4}$ times $\frac{2}{3}$ is $\frac{3}{4}$ of $\frac{2}{3}$, or 3 times $\frac{1}{4}$ of $\frac{2}{3}$, or 3 times $\frac{2}{12}$, which is $\frac{6}{12}$, or $\frac{1}{2}$.

2. How much is $\frac{1}{3}$ times $\frac{3}{4}$? $\frac{2}{3}$ times $\frac{5}{6}$? $\frac{2}{3}$ times $\frac{7}{8}$?
3. How much is $\frac{1}{5}$ times $\frac{2}{3}$? $\frac{2}{5}$ times $\frac{2}{3}$? $\frac{2}{5}$ times $\frac{3}{4}$?
4. How much is $\frac{1}{6}$ times $\frac{3}{5}$? $\frac{5}{6}$ times $\frac{3}{5}$? $\frac{5}{6}$ times $\frac{2}{3}$?

WRITTEN EXERCISES.

5. Multiply $\frac{4}{5}$ by $\frac{3}{4}$.

PROCESS.

$$\frac{4}{5} \times \frac{3}{4} = \frac{4 \times 3}{5 \times 4} = \frac{12}{20} = \frac{3}{5}, \textit{ Ans.}$$

$\frac{3}{4}$ times $\frac{4}{5}$ is $\frac{3}{4}$ of $\frac{4}{5}$, which equals $\frac{12}{20}$, or $\frac{3}{5}$.

Multiply:

6. $\frac{5}{8}$ by $\frac{2}{5}$.
7. $\frac{4}{7}$ by $\frac{7}{6}$.
8. $\frac{3}{15}$ by $\frac{5}{6}$.
9. $\frac{5}{6}$ by $\frac{6}{18}$.
10. $\frac{7}{10}$ by $\frac{7}{12}$.
11. $\frac{18}{16}$ by $\frac{8}{18}$.
12. $\frac{10}{17}$ by $\frac{34}{30}$.
13. $\frac{5}{9}$ by $\frac{9}{20}$.
14. $\frac{9}{25}$ by $\frac{15}{8}$.
15. $2\frac{1}{2}$ by $\frac{5}{6}$.
16. $2\frac{1}{2}$ by $2\frac{1}{2}$.
17. $3\frac{1}{3}$ by $3\frac{1}{3}$.
18. $6\frac{1}{2}$ by $2\frac{1}{3}$.
19. $6\frac{1}{4}$ by $2\frac{1}{2}$.
20. $6\frac{1}{4}$ by $6\frac{1}{4}$.

21. Multiply the multiplicand in each of the above problems from 6 to 15 by $\frac{2}{3}$. By $\frac{3}{5}$.
22. Multiply the multiplier in each of the above problems from 6 to 15 by $\frac{3}{4}$. By $\frac{2}{3}$.

ART. 72. To multiply one fraction by another:

Rule.—*Multiply the numerators together and also the denominators, and reduce the resulting fraction to its lowest terms.*

NOTE.—The process may often be shortened by cancellation.

LESSON XIV.

FRACTIONS DIVIDED BY INTEGERS.

1. A father divided $\frac{3}{4}$ of a melon equally between 3 boys: what part of the melon did each receive?

SOLUTION.—If 3 boys received $\frac{3}{4}$ of a melon, each received $\frac{1}{3}$ of $\frac{3}{4}$ of the melon, which is $\frac{1}{4}$.

2. How much is $\frac{4}{5}$ divided by 3? $\frac{2}{3}$ divided by 3? $\frac{5}{8}$ by 3? $\frac{5}{6}$ by 3? $\frac{5}{8}$ by 3? $\frac{5}{7}$ by 3?

3. How much is $\frac{7}{9}$ divided by 2? $\frac{7}{9}$ by 3? $\frac{7}{9}$ by 7?

4. How much is $\frac{3}{4}$ divided by 5? $\frac{3}{7}$ by 5? $\frac{3}{8}$ by 5? $\frac{3}{11}$ by 5? $\frac{4}{7}$ by 5? $\frac{5}{8}$ by 5? $\frac{7}{9}$ by 5? $\frac{5}{12}$ by 5?

5. How much is $\frac{7}{8} \div 4$? $\frac{7}{8} \div 6$? $\frac{7}{8} \div 10$? $\frac{7}{12} \div 5$?

6. How much is $\frac{5}{11} \div 5$? $\frac{10}{11} \div 4$? $12\frac{1}{2} \div 6$?

SOLUTION.—$12\frac{1}{2}$ divided by 6 is $\frac{1}{6}$ of $12\frac{1}{2}$. $\frac{1}{6}$ of 12 is 2, and $\frac{1}{6}$ of $\frac{1}{2}$ is $\frac{1}{12}$. $12\frac{1}{2}$ divided by 6 is $2\frac{1}{12}$.

7. What is $18\frac{1}{3} \div 6$? $20\frac{2}{3} \div 5$? $30\frac{1}{4} \div 3$?

8. What is $21\frac{2}{3} \div 7$? $25\frac{3}{4} \div 5$? $18\frac{4}{7} \div 6$?

9. What is $13\frac{1}{3} \div 4$? $26\frac{1}{6} \div 6$? $33\frac{2}{3} \div 8$?

WRITTEN EXERCISES.

10. Divide $\frac{17}{20}$ by 3.

PROCESS.

$\frac{17}{20} \div 3 = \frac{17}{20 \times 3} = \frac{17}{60}$, *Ans.* $\frac{17}{20} \div 3$ is $\frac{1}{3}$ of $\frac{17}{20}$, which is $\frac{17}{20 \times 3}$, or $\frac{17}{60}$.

Divide:

11. $\frac{7}{11}$ by 7.
12. $\frac{5}{7}$ by 12.
13. $\frac{7}{12}$ by 10.
14. $\frac{5}{18}$ by 6.
15. $\frac{14}{15}$ by 7.
16. $\frac{16}{17}$ by 8.
17. $\frac{20}{21}$ by 5.
18. $\frac{21}{25}$ by 3.
19. $24\frac{1}{3}$ by 6.
20. $29\frac{3}{5}$ by 7.
21. $46\frac{3}{4}$ by 5.
22. $66\frac{2}{3}$ by 8.

ART. 73. PRINCIPLE.—*A fraction is divided by dividing the numerator or multipling the denominator.*

ART. 74. 1. To divide a fraction by an integer:

Rule.—*Divide the numerator or multiply the denominator.*

2. To divide a mixed number by an integer:

Rule.—*Divide the integral part and then the fraction.*

LESSON XV.

INTEGERS DIVIDED BY FRACTIONS.

1. How many times is $\frac{1}{3}$ of an apple contained in 2 apples? $\frac{2}{3}$ of an apple in 2 apples?

SOLUTION.—2 apples contain 6 thirds of an apple, and 2 thirds are contained in 6 thirds 3 times.

2. How many times is $\frac{3}{4}$ of a yard contained in 3 yards?

3. How many times $\frac{2}{3}$ in 4? $\frac{2}{3}$ in 9? $\frac{2}{3}$ in 10?

4. How many times $\frac{3}{5}$ in 3? $\frac{3}{5}$ in 6? $\frac{3}{5}$ in 6?

5. Divide 12 by $\frac{3}{5}$; 12 by $\frac{3}{4}$; 12 by $\frac{2}{3}$; 12 by $\frac{1}{10}$.

6. Divide 8 by $\frac{4}{7}$; 10 by $\frac{5}{9}$; 10 by $\frac{4}{5}$; 9 by $\frac{7}{9}$.

WRITTEN EXERCISES.

7. Divide 14 by $\frac{3}{5}$.

PROCESS: $14 \div \frac{3}{5} = \frac{70}{5} \div \frac{3}{5} = 70 \div 3 = 23\frac{1}{3}$, *Ans.*

Divide:

8. 25 by $\frac{7}{8}$.	**13.** 60 by $\frac{7}{8}$.	**18.** 40 by $3\frac{1}{3}$.
9. 36 by $\frac{4}{7}$.	**14.** 51 by $\frac{7}{12}$.	**19.** 16 by $6\frac{2}{3}$.
10. 45 by $\frac{5}{9}$.	**15.** 42 by $\frac{7}{10}$.	**20.** 20 by $5\frac{1}{2}$.
11. 63 by $\frac{9}{10}$.	**16.** 56 by $\frac{8}{9}$.	**21.** 28 by $4\frac{1}{2}$.
12. 71 by $\frac{5}{8}$.	**17.** 30 by $2\frac{1}{2}$.	**22.** 33 by $3\frac{2}{3}$.

ART. 75. To divide an integer by a fraction:

Rule.—*Multiply the integer by the denominator of the fraction, and divide the product by the numerator.*

LESSON XVI.

FRACTIONS DIVIDED BY FRACTIONS.

1. How many times is $\frac{1}{4}$ of an orange contained in $\frac{2}{4}$ of it? In $\frac{3}{4}$ of it?

2. How many times $\frac{1}{3}$ of a pear contained in $\frac{2}{3}$ of it?

3. How many times $\frac{1}{8}$ in $\frac{2}{8}$? $\frac{1}{8}$ in $\frac{3}{8}$? $\frac{1}{4}$ in $\frac{3}{4}$?
4. How many times $\frac{2}{5}$ in $\frac{4}{5}$? $\frac{2}{5}$ in $\frac{6}{5}$? $\frac{2}{5}$ in $\frac{10}{5}$?
5. How many times $\frac{1}{4}$ in $\frac{1}{2}$? $\frac{1}{4}$ in $\frac{3}{2}$? $\frac{3}{4}$ in $\frac{3}{2}$?

SUGGESTION.—Reduce the fractions to a common denominator.

6. How many times $\frac{1}{8}$ in $\frac{1}{2}$? $\frac{2}{8}$ in $\frac{4}{2}$? $\frac{2}{5}$ in $\frac{5}{4}$?
7. Divide $\frac{3}{4}$ by $\frac{2}{3}$. $\frac{2}{8}$ by $\frac{3}{4}$. $\frac{2}{5}$ by $\frac{1}{2}$. $\frac{1}{2}$ by $\frac{2}{3}$.

WRITTEN EXERCISES.

8. Divide $\frac{5}{7}$ by $\frac{2}{3}$.

PROCESS.

$\frac{5}{7} = \frac{15}{21}$; $\frac{2}{3} = \frac{14}{21}$ $\quad \frac{15}{21} \div \frac{14}{21} = 15 \div 14 = 1\frac{1}{14}$, *Ans.*

Divide:

9. $\frac{5}{12}$ by $\frac{5}{8}$.
10. $\frac{6}{7}$ by $\frac{3}{4}$.
11. $\frac{5}{8}$ by $\frac{5}{6}$.
12. $\frac{7}{8}$ by $\frac{3}{4}$.
13. $\frac{3}{5}$ by $\frac{1}{2}$.
14. $\frac{3}{12}$ by $\frac{5}{8}$.
15. $\frac{6}{7}$ by $\frac{2}{3}$.
16. $\frac{5}{9}$ by $\frac{2}{3}$.
17. $\frac{5}{6}$ by $\frac{3}{8}$.
18. $2\frac{1}{2}$ by $\frac{2}{3}$.
19. $3\frac{1}{3}$ by $\frac{3}{5}$.
20. $\frac{3}{5}$ by $2\frac{1}{2}$.
21. $\frac{5}{8}$ by $\frac{3}{4}$.
22. $\frac{3}{4}$ by $\frac{3}{8}$.
23. $\frac{3}{5}$ by $\frac{7}{8}$.
24. $\frac{7}{8}$ by $\frac{3}{5}$.
25. $\frac{2}{3}$ by $\frac{5}{6}$.
26. $\frac{5}{6}$ by $\frac{2}{3}$.
27. $\frac{7}{8}$ by $\frac{5}{7}$.
28. $\frac{3}{5}$ by $\frac{7}{9}$.
29. $\frac{5}{12}$ by $\frac{3}{4}$.
30. $\frac{3}{4}$ by $\frac{5}{12}$.
31. 8 by $\frac{5}{6}$.
32. 9 by $\frac{6}{7}$.

33. Divide $\frac{5}{7}$ by $\frac{2}{3}$ by inverting the terms of the divisor.

PROCESS: $\frac{5}{7} \div \frac{2}{3} = \frac{5 \times 3}{7 \times 2} = \frac{15}{14} = 1\frac{1}{14}$, *Ans.*

The above process is a *short method* of reducing the fractions to a common denominator and then dividing their numerators. This may be shown as follows:

$\frac{5}{7} = \frac{5 \times 3}{21}$; $\frac{2}{3} = \frac{7 \times 2}{21}$ $\quad \frac{5 \times 3}{21} \div \frac{7 \times 2}{21} = \frac{5 \times 3}{7 \times 2} = \frac{15}{14} = 1\frac{1}{14}$.

NOTE.—Only one explanation of this process should be given to young pupils. For another explanation, see "New Complete Arithmetic."

34. Solve each of the above problems, from 9 to 33, by inverting the terms of the divisor.

35. Divide the dividend in each of the above problems (9 to 33) by $\frac{3}{4}$. By $\frac{2}{3}$.

ART. 76. To divide a fraction by a fraction:

Rule.—*Reduce the fractions to equivalent fractions with a common denominator, and divide the numerator of the dividend by the numerator of the divisor.* Or:

Invert the terms of the divisor, and then multiply the numerators together, and also the denominators, and reduce the resulting fraction to its lowest terms.

LESSON XVII.

MISCELLANEOUS PROBLEMS.

ORAL PROBLEMS.

1. How much is $\frac{5}{8}$ of 56? $\frac{5}{7}$ of 50?

2. How much is $\frac{1}{3}$ of $12\frac{1}{2}$? $\frac{1}{5}$ of $62\frac{1}{2}$?

3. $16\frac{2}{3}$ is $\frac{1}{6}$ of what number?

4. 27 is $\frac{3}{7}$ of what number?

5. 35 is $\frac{4}{5}$ of what number?

6. 62 feet are $\frac{5}{8}$ of how many feet?

7. Clara is 12 years old, and her age is $\frac{2}{5}$ of her father's age: how old is her father?

8. Edward is 21 years old, and his age is $\frac{3}{7}$ of his mother's age: how old is his mother?

9. A man spent $\frac{2}{5}$ of his money and had $21 left: how much money had he at first?

10. A boy gave 24 cents for a slate, which was $\frac{4}{7}$ of his money: how much money did he have?

11. A man pays $25 a month for house-rent, which is $\frac{5}{12}$ of his monthly salary: what is his salary?

12. A farmer sold a cow for $45, which was $\frac{1}{4}$ more than he paid for her: what was the cost of the cow?

13. A farmer has 24 acres of woodland, which is $\frac{4}{15}$ of his farm: how many acres in his farm?

14. $\frac{1}{4}$ of a farm is in wheat, $\frac{1}{3}$ of it in corn, and the rest in grass: what part is in grass?

15. A boy spent $\frac{1}{3}$ of his money for a sled, and $\frac{2}{5}$ of it for a pair of skates: what part had he left?

16. John bought a knife for $\frac{3}{5}$ of a dollar and a ball for $\frac{1}{4}$ of a dollar, and then sold both of them for a dollar: what part of a dollar did he gain?

17. Jane having a quart of plums, gave $\frac{1}{3}$ of a quart to her brother and $\frac{1}{4}$ of a quart to her sister: what part of a quart had she left?

18. A farmer bought 2 bushels of clover-seed, and then sold $\frac{2}{5}$ of a bushel to one neighbor and $\frac{3}{4}$ of a bushel to another: what part of a bushel had he left?

19. A student spends $\frac{2}{5}$ of his time in study, $\frac{1}{10}$ of it in labor, and $\frac{1}{3}$ of it in sleep: what part has he left?

20. One sixth of a pole is in the ground, $\frac{2}{5}$ of it in water, and the rest in the air: what part is in the air?

21. A man sold $\frac{2}{5}$ of his farm to one neighbor and $\frac{3}{8}$ of it to another: what part of the farm has he left?

22. A boy having $\frac{3}{4}$ of a melon, gave $\frac{1}{2}$ of it to a playmate: what part did the playmate receive?

23. A woman divided $\frac{4}{5}$ of a pound of crackers equally between 3 poor children: what part of a pound did each receive?

24. If 6 men can build $\frac{7}{8}$ of a wall in a day, what part of the wall can one man build?

25. If 5 pounds of cheese cost $\$\frac{3}{4}$, what will 1 pound cost?

26. A grocer put $16\frac{1}{2}$ pounds of sugar into 5 equal parcels: how much sugar was put into each parcel?

27. If a basket hold $\frac{2}{5}$ of a bushel, how many baskets will hold 4 bushels?

28. If a pound of butter cost $\$\frac{1}{4}$, how many pounds can be bought for $\$2\frac{1}{2}$?

29. If $\frac{2}{5}$ of a yard of cloth will make a vest, how many vests will $1\frac{4}{5}$ yards make?

30. A man owning $\frac{5}{8}$ of a factory sold $\frac{2}{5}$ of his share: what part of the factory did he sell? What part does he still own?

WRITTEN PROBLEMS.

31. Reduce $18\frac{3}{4}$ to an improper fraction.

32. Reduce $\frac{113}{10}$ to a mixed number.

33. Reduce $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator.

34. Add $\frac{1}{3}$, $\frac{1}{6}$, $\frac{3}{4}$, and $\frac{5}{12}$.

35. Add $28\frac{3}{5}$, $40\frac{1}{2}$, $63\frac{2}{3}$, and $19\frac{3}{10}$.

36. From $28\frac{2}{5}$ take $16\frac{2}{3}$.

37. Multiply $\frac{5}{8}$ by 7; 18 by $\frac{3}{5}$; $\frac{3}{4}$ by $\frac{6}{7}$.

38. Multiply $187\frac{2}{3}$ by 15; 256 by $21\frac{5}{8}$.

39. Divide 12 by $\frac{2}{3}$; $\frac{2}{3}$ by 12; $\frac{3}{5}$ by $\frac{6}{7}$.

40. Divide $243\frac{7}{8}$ by 11; 256 by $5\frac{1}{2}$.

41. $\frac{7}{8} + \frac{2}{3} =$ what? $\frac{7}{8} - \frac{2}{3}$? $\frac{7}{8} \times \frac{2}{3}$? $\frac{7}{8} \div \frac{2}{3}$?

42. There are 5280 feet in a mile: how many feet in $\frac{9}{10}$ of a mile? In $\frac{5}{16}$ of a mile?

43. A vessel is worth \$6000, and the cargo is worth $\frac{5}{8}$ as much as the vessel: what is the value of the cargo?

44. There are $16\frac{1}{2}$ feet in a rod: how many feet in 66 rods? How many rods in 2640 feet?

45. There are $5\frac{1}{2}$ yards in a rod: how many rods in 66 yards? In 440 yards?

46. At $\$6\frac{1}{4}$ a barrel, how many barrels of flour can be bought for \$150? For \$225?

47. If $\frac{4}{7}$ of a ship is worth \$12000, what is the whole ship worth?

48. A man sold $\frac{3}{7}$ of a farm for \$1500: at this rate, what was the value of the farm?

49. A farmer had 2 fields of wheat; the first yielded 840 bushels, which was $\frac{5}{12}$ of the amount yielded by the second field: how many bushels did the second field yield?

50. Two men bought a newspaper for \$3800, one paying $\frac{2}{3}$ of the amount and the other $\frac{1}{3}$: how much did each pay?

51. A farmer bought $240\frac{5}{8}$ acres of land, and sold $90\frac{3}{8}$ acres and $75\frac{1}{4}$ acres: how many acres had he left?

52. From a piece of broadcloth containing $20\frac{3}{4}$ yards, a merchant sold $5\frac{1}{2}$ yards, $4\frac{1}{8}$ yards, and $8\frac{1}{4}$ yards: how many yards were left?

53. A man earned $\$56\frac{3}{8}$ one month and $\$70\frac{5}{8}$ the next, and then gave $\$85\frac{1}{8}$ for a horse: how much money had he left?

54. A man owning $\frac{2}{3}\frac{1}{2}$ of a vessel, sold $\frac{1}{4}$ and $\frac{1}{8}$ of the vessel: what part had he left? If this part is worth \$9,000, what is the value of the vessel?

55. A pedestrian walked $\frac{4}{15}$ of his journey the first day, $\frac{5}{12}$ of it the next day, and completed it the third day: what part of the journey did he travel the third day? If he walked 19 miles the third day, how long was the journey?

56. A man sold $\frac{1}{4}$ of his farm to A, $\frac{1}{3}$ of it to B, and the rest, which was 120 acres, to C: what part of the farm did he sell to C? How many acres in the farm?

57. A man bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of it to each of his two sons, and the rest, which was \$2400, to his daughter: what part did the daughter receive? What was the value of the estate?

58. A fruit dealer bought 80 barrels of apples, each containing $2\frac{3}{4}$ bushels, at $\$2\frac{1}{2}$ per barrel, and then sold the apples at $\$1\frac{1}{5}$ a bushel: how much did he gain?

59. How many pounds of cheese, at $12\frac{1}{2}$ cents a pound, can be bought for $65\frac{3}{4}$ pounds of butter at 25 cents a pound?

60. Two men start from the same place and walk in opposite directions, one $2\frac{7}{8}$ miles per hour, and the other $3\frac{1}{5}$ miles per hour: how far will they be apart in 1 hour? In $6\frac{1}{4}$ hours?

61. A man owning $\frac{8}{9}$ of a ship sells $\frac{2}{3}$ of his share for \$4400: at this rate, what is the value of the ship?

62. A grocer bought 4 crocks of butter, weighing respectively $31\frac{3}{8}$ lb., $32\frac{5}{16}$ lb., $27\frac{1}{16}$ lb., and $30\frac{3}{4}$ lb., and the weight of the empty crocks were $5\frac{3}{8}$ lb., $5\frac{5}{16}$ lb., $6\frac{1}{4}$ lb., and $5\frac{1}{16}$ lb.: what was the weight of the butter? What was its cost at $33\frac{1}{3}$ cts. a pound?

63. How many bushels of wheat, allowing 60 pounds to the bushel, are in 6 sacks, weighing respectively $125\frac{7}{16}$ lb., $130\frac{3}{8}$ lb., $128\frac{3}{4}$ lb., 132 lb., $133\frac{1}{2}$ lb., and $138\frac{3}{8}$ lb., the weight of the six empty sacks together being $8\frac{7}{16}$ lb.?

QUESTIONS FOR REVIEW.

What is a fraction? What does the denominator denote? The numerator? What are the terms of a fraction?

When is a fraction called proper? When improper? When is the value of an improper fraction equal to 1? What is a mixed number? What is meant by $18\frac{3}{4}$? *Ans.*, $18 + \frac{3}{4}$.

How is an integer reduced to a fraction? How is a mixed number reduced to a fraction? What kind of a fraction is the result? Give examples. How is an improper fraction reduced to an integer or mixed number?

How is a fraction reduced to lower terms? On what principle does the process depend? How may a fraction be reduced to its *lowest* terms by one division?

How are fractions having a common denominator added or subtracted? When fractions have different denominators, how are they added or subtracted? How may mixed numbers be added? How may they be subtracted?

In what two ways may the fractional part of an integer be found?

What is a simple fraction? What is a compound fraction? How may a compound fraction be reduced to a simple fraction?

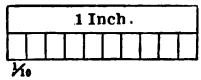
In what two ways may a fraction be multiplied by an integer? How may an integer be multiplied by a fraction? Give the rule for multiplying a fraction by a fraction.

In what two ways may a fraction be divided by an integer? How may an integer be divided by a fraction? How may a fraction be divided by a fraction?

DECIMAL FRACTIONS.

LESSON XVIII.

NUMERATION AND NOTATION.



1. If an inch be divided into ten equal parts, what is one part called?

2. If a tenth of an inch be divided into ten equal parts, what is one part? What is $\frac{1}{10}$ of $\frac{1}{10}$?

3. If a hundredth of an inch be divided into ten equal parts, what is one part? What is $\frac{1}{10}$ of $\frac{1}{100}$?

4. What is one tenth of one tenth? ($\frac{1}{10}$ of $\frac{1}{10}$?) One tenth of one hundredth? ($\frac{1}{10}$ of $\frac{1}{100}$?)

5. How many thousandths make one hundredth? How many hundredths make one tenth? How many tenths make a unit, or one?

ART. 77. Such parts of a unit as one tenth, one hundredth, one thousandth, etc., are *decimal fractions*, since they arise from a *decimal* division of a unit.

Since 10 thousandths equal 1 hundredth, 10 hundredths 1 tenth, 10 tenths 1 unit, such decimal fractions may be expressed, like integers, *on a scale of ten*. This is done by writing the tenths in the first order at the right of units, the hundredths in the second order, the thousandths in the third order, etc., and placing a period between the orders of units and tenths, to distinguish the fractional orders from the integral.

Thus, $2\frac{5}{10}$ is written 2.5; $12\frac{24}{100}$ is written 12.24; and $22\frac{3}{1000}$ is written 22.003.

6. How many tenths in .3? .6? .7? .8? .9?

7. How many hundredths in .02? In .04? .07?

8. How many tenths and hundredths in .24? .36?

9. How many thousandths in .005? .008? .006?

10. How many tenths, hundredths, and thousandths in .356? In .523? .603? .041? .406? .007?

ART. 78. Two tenths and five hundredths (.25) denote twenty-five *hundredths;* and two hundredths and five thousandths (.025) denote twenty-five *thousandths.*

11. How many hundredths in .34? .45? .57? .06? .09? .69? .07? .57? .08? .78?

12. How many thousandths in .246? .048? .007? .403? .059? .097? .246? .307? .184?

ART. 79. When fractions denoting tenths, hundredths, thousandths, etc., are expressed on a scale of ten, they are said to be expressed *decimally,* and are called *decimals.*

The decimal orders are numbered from left to right, the first decimal order being *tenths;* the second, *hundredths;* the third, *thousandths,* etc.

13. Read the decimals .45; 2.6; .035; .03; .005.

14. Read the decimals .34; 25.03; 2.015; 15.045.

Read the following decimals:

15.	**16.**	**17.**	**18.**	**19.**
.12	.014	.324	2.53	.004
.34	.063	.406	12.6	.803
.06	.008	.065	7.009	.55
.5	.030	.704	8.057	.4

WRITTEN EXERCISES.

Express decimally the following fractions:

20.	**21.**	**22.**	**23.**	**24.**
$\frac{3}{10}$	$\frac{25}{100}$	$\frac{225}{1000}$	$\frac{6}{10}$	$2\frac{3}{10}$
$\frac{5}{10}$	$\frac{6}{100}$	$\frac{305}{1000}$	$\frac{6}{100}$	$\frac{23}{100}$
$\frac{7}{10}$	$\frac{31}{100}$	$\frac{7}{1000}$	$\frac{6}{1000}$	$\frac{23}{1000}$

25.	**26.**	**27.**
25 hundredths.	$26\frac{1}{8}$ hundredths.	18 hundredths.
7 hundredths.	$6\frac{1}{8}$ thousandths.	204 thousandths.
205 thousandths.	28 thousandths.	50 thousandths.
68 hundredths.	624 thousandths.	300 thousandths.
68 thousandths.	97 hundredths.	40 hundredths.
405 thousandths.	106 thousandths.	405 thousandths.

28. Sixty-three thousandths.
29. Eight units and seven tenths.
30. Seven hundred eight thousandths.
31. Four hundred twelve thousandths.
32. Fifteen units and twelve hundredths.

ART. 80. The fourth decimal order is called *ten-thousandths;* the fifth, *hundred-thousandths;* and the sixth, *millionths.*

Copy and read:

33.	**34.**	**35.**	**36.**
.445	.0304	.3256	.00267
.0445	.00304	.4048	.000267
.706	.475	.03256	.004324
.0706	.00475	.04048	.046375

Express as decimals:

37.	**38.**	**39.**	**40.**
$\frac{2124}{10000}$	$\frac{23462}{100000}$	$\frac{824562}{1000000}$	$\frac{405}{10000}$
$\frac{324}{10000}$	$\frac{236}{100000}$	$\frac{4078}{1000000}$	$\frac{405}{1000000}$
$\frac{24}{10000}$	$\frac{75}{100000}$	$\frac{605}{10000000}$	$\frac{405}{100000}$
$\frac{8}{10000}$	$\frac{7}{100000}$	$\frac{67}{1000000}$	$\frac{405}{1000}$

41. Four hundred twenty-two millionths.
42. Seven hundred twelve ten-thousandths.
43. Fifteen millionths.
44. Four hundred fourteen hundred-thousandths.
45. Two hundred seventeen ten-thousandths.

ART. 81. An integer and a decimal, written together as one number, are connected by *and* when expressed in words. Thus, 45.14 is read *forty-five units and fourteen hundredths*, or, more briefly, *forty-five and fourteen hundredths.*

Write in words:

46.	47.	48.
27.305	2.00304	300.006
63.0346	20.0304	400.0012
504.3507	200.304	5000.004

49. Express decimally forty-five units and fifty-two hundredths.

50. Forty units and forty-five thousandths.

51. Two hundred units and seventy-nine millionths.

52. Two hundred seventy-nine millionths.

53. Five thousand units and sixty-six millionths.

54. Five thousand sixty-six millionths.

55. Fourteen units and five thousand fourteen hundred-thousandths.

56. One thousand eighty-seven millionths.

57. Eight hundred forty-two ten-thousandths.

58. Seventy-five units and four hundred three hundred-thousandths.

See "Manual of Arithmetic" for additional examples.

DEFINITIONS, PRINCIPLES, AND RULES.

ART. 82. A **Decimal Fraction** is one or more of the decimal parts of a unit.

It results from the division of a unit into tenths, tenths into hundredths, hundredths into thousandths, etc., called the *decimal* division.

The denominator of a decimal fraction is 10, 100, 1000, 10000, etc.

ART. 83. Decimal fractions may be expressed in three ways:

1. By words; as, three tenths, twelve hundredths, one hundred ninety-six thousandths.

2. By writing the denominator under the numerator; as, $\frac{3}{10}$, $\frac{12}{100}$, $\frac{196}{1000}$.

3. By omitting the denominator and writing the numerator in the decimal form; as, .3, .12, .196

NOTE.—When a decimal fraction is written with a decimal point and without its denominator, as .25, it is written in the *decimal form*, so called because only decimal fractions can be thus expressed. When a decimal fraction is written in the decimal form, it is called a *decimal*. See "New Complete Arithmetic."

ART. 84. The **Decimal Point** is a period placed at the left of the order of tenths, to designate the decimal orders.

ART. 85. A **Mixed Decimal Number** is an integer and a decimal written together as one number; as, 2.45. It is also called a *Mixed Number*.

The orders on the left of the decimal point are *integral*, and those on the right are *decimal*. The decimal orders are called *Decimal Places*.

ART. 86. The following table gives the names of six integral and six decimal orders:

Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
6	5	4	3	2	1	.	1	2	3	4	5	6
Integral Orders.							Decimal Orders.					

ART. 87. PRINCIPLES.—1. *The successive decimal orders decrease in value from left to right, and increase from right to left, in the same manner as integral numbers.* Hence,

2. *The removal of a decimal figure one order to the left multiplies its value by 10, and its removal one order to the right divides its value by 10.*

ART. 88. To read a decimal:

Rule.—*Read it as if it were an integer, and add the name of the right-hand order.*

NOTES.—1. A decimal is read precisely as it would be, were the denominator expressed.

2. In reading a mixed decimal number, the word "units" should be supplied if its omission would render the meaning ambiguous. 300.006 should be read *three hundred units and six thousandths.*

3. While "and" must be used to connect the integral and decimal parts of a mixed decimal number, it may also be used to connect the parts of an integer (Art. 20, note 2) and also of a pure decimal. Thus, .306 may be read *three hundred six thousandths* or *three hundred and six thousandths.* The latter is common usage, but the former is preferred by many teachers.

ART. 89. To write a decimal:

Rule.—*Write the numerator and so place the decimal point that the right-hand figure shall stand in the order denoted by the name of the decimal.*

NOTES.—1. The name of a decimal is the same as the name of its right-hand order.

2. Pupils should be taught to write decimals by first making the decimal point, and then filling the successive orders from left to right, with ciphers or significant figures, as may be required. Suppose, for example, that the decimal to be written is 2312 millionths. Since millionths is the *sixth* decimal order, and the numerator of the decimal contains only four figures, it is seen that there will be two orders to fill with ciphers, thus: .002312.

LESSON XIX.

REDUCTION OF DECIMALS.

I. DECIMALS REDUCED TO HIGHER OR LOWER ORDERS.

1. How many hundredths in 1 tenth? In 3 tenths?
2. How many thousandths in 1 hundredth? .12?
3. How many tenths in 10 hundredths? In 40?
4. How many hundredths in 10 thousandths? .040?

WRITTEN EXERCISES.

5. Reduce .325 to hundred-thousandths.

PROCESS: $.325 = .32500$, since $\frac{325}{1000} = \frac{32500}{100000}$.

6. Reduce .45 to ten-thousandths.
7. Reduce 6.5 to thousandths.
8. Reduce 23 to hundredths.
9. Reduce 62.5 to ten-thousandths.
10. Reduce .4500 to hundredths.

PROCESS: $.4500 = .45$, *Ans.*

11. Reduce .5000 to tenths.
12. Reduce 2.4000 to hundredths.

ART. 90. PRINCIPLES.—1. *Annexing ciphers to a decimal, or decimal ciphers to an integer, does not change its value.*

2. *Removing ciphers from the right of a decimal, or decimal ciphers from the right of an integer, does not change its value.*

II. DECIMALS REDUCED TO COMMON FRACTIONS.

13. Reduce .225 to a common fraction in its lowest terms.

PROCESS: $.225 = \frac{225}{1000} = \frac{9}{40}$, *Ans.*

Reduce to a common fraction in lowest terms:

14. .75	**19.** .035	**24.** .0096	**29.** .0125
15. .625	**20.** .075	**25.** 3.25	**30.** .0032
16. .0625	**21.** .024	**26.** 21.075	**31.** 12.375
17. .125	**22.** .512	**27.** 4.375	**32.** 25.032
18. .08	**23.** .275	**28.** 18.75	**33.** 2.0625

ART. 91. To reduce a decimal to a common fraction in its lowest terms:

Rule.—*Omit the decimal point and supply the denominator, and then reduce the fraction to its lowest terms.*

III. COMMON FRACTIONS REDUCED TO DECIMALS.

34. How many tenths in $\frac{1}{2}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

35. How many hundredths in $\frac{4}{25}$? $\frac{8}{25}$? $\frac{6}{25}$?

WRITTEN EXERCISES.

36. Reduce $\frac{8}{25}$ to a decimal.

SOLUTION.—$\frac{8}{25} = \frac{1}{25}$ of $8.00 = .32$

37. Reduce $\frac{5}{8}$ to a decimal.

WRITTEN PROCESS.

25)8.00(.32, *Ans.*
75
50
50

Reduce to a decimal:

38. $\frac{3}{20}$.	**42.** $\frac{2}{25}$.	**46.** $\frac{3}{400}$.	**50.** $56\frac{1}{4}$.	**54.** $.62\frac{1}{2}$.
39. $\frac{1}{4}$.	**43.** $\frac{3}{40}$.	**47.** $\frac{8}{50}$.	**51.** $13\frac{2}{5}$.	**55.** $.12\frac{2}{25}$.
40. $\frac{3}{8}$.	**44.** $\frac{5}{80}$.	**48.** $\frac{3}{200}$.	**52.** $30\frac{3}{4}$.	**56.** $4.06\frac{1}{4}$.
41. $\frac{1}{16}$.	**45.** $\frac{32}{125}$.	**49.** $\frac{8}{125}$.	**53.** $7\frac{2}{25}$.	**57.** $15.18\frac{3}{4}$.

ART. 92. To reduce a common fraction to a decimal:

Rule.—*Annex decimal ciphers to the numerator, then divide by the denominator, and point off as many decimal places in the quotient as there are annexed ciphers.*

LESSON XX.

ADDITION OF DECIMALS.

1. What is the sum of 5 tenths and 4 tenths? 6 tenths and 9 tenths?

SOLUTION.—6 tenths and 9 tenths are 15 tenths, which is equal to 1 unit and 5 tenths.

2. What is the sum of 8 hundredths and 7 hundredths? 18 hundredths and 7 hundredths?

3. What is the sum of 28 thousandths and 9 thousandths? 56 thousandths and 8 thousandths?

WRITTEN EXERCISES.

4. What is the sum of 24.6, 307.08, 93.609, .456, 400.06, 37.027?

PROCESS.

24.6	Since only like orders can be added, write the figures of the same order in the same column. Since ten units of any order equal 1 unit of the next left-hand order, begin at the right, and add as in simple numbers. Place the decimal point at the left of the tenths' order in the amount.
307.08	
93.609	
.456	
400.06	
37.027	

862.832, *Ans.*

5. What is the sum of .4506, .709, and 27.0508?

6. Add 15.34, 6.078, 60.804, and 99.875

7. Add \$21.94, \$87.075, \$9.858, and \$807.62½

8. What is the sum of 47.6¼ miles, 19.48 miles, 34.75 miles, and 76.62½ miles?

9. Add thirty-nine hundredths; six hundred eight ten-thousandths; and eighty-seven thousandths.

10. Add forty-seven thousandths; forty-seven millionths; sixteen ten-thousandths; and sixteen hundredths.

ART. 93. To add decimals:

Rule.—1. *Write the numbers so that figures of the same order shall be in the same column.*

2. *Beginning at the right, add as in the addition of integers, and place the decimal point at the left of the tenths' order in the amount.*

LESSON XXI.

SUBTRACTION OF DECIMALS.

WRITTEN EXERCISES.

1. From 56.403 take 18.6.

PROCESS:
56.403
18.6
37.803, *Ans.*

Since only like units can be subtracted, write the numbers so that figures of the same order shall be in the same column.

Since ten units of any decimal order make one unit of the next left-hand order, subtract as in integers, and place the decimal point at the left of the tenths' order in the remainder.

2. From 56.6 take 18.403.

PROCESS:
56.600
18.403
38.197, *Ans.*

3. From 45.3 take 28.756.

4. From .0407 take .008075.

5. From twelve thousandths take eight millionths.

6. From eight tenths take eight ten-thousandths.

7. From $47.065 + 36.87$ take $9.08 + 43.375$.

8. From $5.25 + .087$ take $5.25 - .087$.

9. From the sum of twenty-five thousandths and forty-six ten-thousandths take their difference.

ART. 94. To subtract decimals:

Rule.—1. *Write the numbers so that figures of the same order shall be in the same column.*

2. *Subtract as in the subtraction of integers, and place the decimal point at the left of the tenths' order in the remainder.*

LESSON XXII.

MULTIPLICATION OF DECIMALS.

1. How much is 3 times 2 tenths? 3 times $\frac{5}{10}$?
2. How much is 4 times 3 tenths? 4 times .4?
3. How much is 5 times $\frac{3}{100}$? 5 times .05?
4. How much is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ by $\frac{1}{100}$? $\frac{1}{10} \times \frac{1}{1000}$?
5. How much is $\frac{1}{100}$ of $\frac{1}{10}$? $\frac{1}{100} \times \frac{1}{10}$? $\frac{1}{100} \times \frac{1}{1000}$?
6. What is the product when tenths are multiplied by tenths? Tenths by hundredths? Hundredths by hundredths? Hundredths by thousandths?

WRITTEN EXERCISES.

7. Multiply .435 by .65.

PROCESS.

.435
.65
2175
2610
.28275, *Ans.*

Since thousandths multiplied by hundredths produce *hundred-thousandths* ($\frac{435}{1000} \times \frac{65}{100} = \frac{28275}{100000}$), the product of .425 and .65 contains *five* decimal orders, or *as many as both of the factors.*

8. Multiply fifteen thousandths by three hundredths.
9. Multiply forty-six ten-thousandths by six and four tenths.
10. Multiply forty-seven thousandths by two and four hundredths.

Multiply:

11. .347 by .73; by 7.3
12. .48 by 3.6; by .36
13. .067 by .65; by 6.5
14. 3.42 by 5.4; by .054
15. 47.5 by .034; by 3.4
16. 492 by 30.6; by 3.06
17. 650 by 2.4; by .0024
18. 30.3 by .044
19. .008 by .007
20. .075 by .48$\frac{1}{3}$
21. 2.42 by 50.9
22. .0075 by 2.8
23. 43.6 by .073
24. .024 by .06$\frac{1}{2}$

25. Multiply the multiplicand in each of the above problems, from 11 to 24, by .035. By 2.06

26. Multiply 4.35 by 10; by 100.

PROCESS.

$4.35 \times 10 = 43.5$, *Ans.*
$4.35 \times 100 = 435.$, *Ans.*

Since the removal of a decimal figure one place to the left multiplies its value by 10 (Art. 9, Pr. 4), the removal of the decimal point one place to the right (which removes each figure one place to the left) multiplies 4.35 by 10. The removal of the decimal point two places to the right multiplies 4.35 by 100.

27. Multiply 4.085 by 100; by 1000; by 10.

28. Multiply 3.0048 by 100; by 1000; by 10000.

PRINCIPLES AND RULES.

ART. 95. PRINCIPLES.—1. *The number of decimal places in the product equals the number of decimal places in both factors.*

2. *Each removal of the decimal point one place to the right multiplies a decimal by 10.*

ART. 96. 1. To multiply one decimal by another:

Rule.—*Multiply as in the multiplication of integers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.*

NOTE.—If there be not enough decimal figures in the product, supply the deficiency by prefixing decimal ciphers.

2. To multiply a decimal by 10, 100, 1000, etc.:

Rule.—*Remove the decimal point as many places to the right as there are ciphers in the multiplier.*

NOTE.—If there be not enough decimal places in the product, supply the deficiency by annexing ciphers.

LESSON XXIII.

DIVISION OF DECIMALS.

1. How many times are 2 tenths contained in 8 tenths? 3 tenths in 9 tenths?

2. How many times are 4 hundredths contained in 12 hundredths? 6 hundredths in 42 hundredths?

3. How many times 6 thousandths in 24 thousandths?

4. Divide 18 hundredths by 6 hundredths; 30 thousandths by 5 thousandths.

5. Of what order is the quotient when tenths are divided by tenths? Hundredths by hundredths?

6. Of what order is the quotient when any number is divided by a like number?

WRITTEN EXERCISES.

7. Divide 6.25 by .25

PROCESS.

```
.25)6.25(25
   50
   125
   125
```

Since 25 hundredths are contained in 625 hundredths (a like number) 25 times, the quotient is an integer.

8. Divide .625 by .25

PROCESS.

```
.25).625(2.5
   50
   125
   125
```

Since the divisor (.25) and the first partial dividend (.62) are both hundredths (like numbers), the first quotient figure is *units*, and hence the second is *tenths*.

9. Divide 17.28 by .48; by 4.8; by .16

10. Divide 3.528 by .042; by 12.6; by 2.52

11. Divide .9408 by 8.4; by .084; by 16.8

12. Divide .06241 by 79; by .079; by 2.37.
13. Divide 18.816 by 1.68; by 168.; by 3.36
14. Divide 17.595 by .85; by 2.07; by 25.5
15. Divide .0768 by 9.6; by .096; by 2.88

16. Divide 62.5 by .025

PROCESS.

.025)62.500(2500., *Ans.*
50
125
125
00

By annexing two decimal-ciphers to 62.5, the dividend and divisor are made like numbers, and hence their quotient is an integer.

17. Divide 25.6 by .032; by .16; by .048
18. Divide 2.5 by 1.25; by .0125; by .021
19. Divide 45.3 by 3.02; by .0302; by .604
20. Divide 80.5 by .35; by .00035; by .0115
21. Divide 402.5 by 1.75; by .0175; by .00035
22. Divide 86.075 by 27.5; by .055; by .0275
23. Divide the dividend in each of the above problems, from 9 to 23, by .025; by 2.5

24. Divide 34.5 by 10; by 100.

PROCESS.

$34.5 \div 10 = 3.45$
$34.5 \div 100 = .345$

The removal of the decimal point one place to the left, removes each figure in 34.5 one place to the right, and hence divides the number by 10; and the removal of the decimal point two places to the left divides it by 100.

25. Divide 436.7 by 100; by 1000; by 10000.
26. Divide 234.6 by 100; by 1000; by 100000.

PRINCIPLES AND RULES.

ART. 97. PRINCIPLES.—1. *The dividend contains as many decimal places as both divisor and quotient.*

2. *The quotient contains as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.*

3. *Each removal of the decimal point one place to the left divides a decimal by 10.*

ART. 98. 1. To divide one decimal by another:

Rule.—*Divide as in the division of integers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

NOTES.—1. When the divisor contains more decimal places than the dividend, supply the deficiency in the dividend by *annexing* decimal ciphers.

2. When the quotient has not enough decimal figures, supply the deficiency by *prefixing* decimal ciphers.

3. When there is a remainder, the division may be continued by annexing ciphers, each cipher thus annexed adding one decimal place to the dividend. Sufficient accuracy is usually secured by carrying the division to three decimal places.

2. To divide a decimal by 10, 100, 1000, etc.:

Rule.—*Remove the decimal point as many places to the left as there are ciphers in the divisor.*

LESSON XXIV.

REVIEW PROBLEMS.

1. Express decimally $2\frac{1}{8}$ hundredths.
2. Express decimally $16\frac{1}{8}$ thousandths.
3. Change .0325 to a common fraction.
4. Change $\frac{8}{125}$ to a decimal.
5. Change $16\frac{3}{40}$ to a mixed decimal.
6. From $56.675 + 48.12\frac{1}{2}$ take 5.0875
7. Add 3.47, 14.086, .583, .087, and 42.3

8. Multiply $62.5 + 37.564$ by 2.55

9. Divide 84.564 by 9.72; by .0012

10. Divide .3672 by .036; by 360.

11. Divide $25.6 \times .56$ by .0128; by 420.

12. Divide 348.6 by 100, and the quotient by 20.

13. Divide 17.28 by .144; by 2400.

14. Multiply $27.5 \div .025$ by $76.8 \div .48$

15. If a man travel 3.25 miles in an hour, how many miles will he travel in 12.4 hours?

16. If a steamer sail 12.5 miles per hour, in how many hours will it sail 102.5 miles?

17. If 3.5 acres produce 141.4 bushels of corn, what is the yield per acre?

18. The product of two numbers is 6.23, and one of the numbers is 124.6: what is the other number?

QUESTIONS FOR REVIEW.

What is a decimal fraction? What is meant by the decimal division of a unit? What is the denominator of a decimal fraction?

In how many and what ways may a decimal fraction be expressed? When is it usually called a *decimal?* What is a mixed decimal number or mixed decimal?

What is the decimal point? What is its use? What is the first decimal order called? The second? Third? Fourth? Fifth? Sixth? By what other name are decimal orders called?

Does the value of the decimal orders increase or decrease from left to right? From right to left? What is the name of a decimal?

Give the rule for reading decimals. Give the rule for writing decimals. How is the value of a decimal affected by annexing ciphers? By removing ciphers from the right?

Give the rule for reducing decimals to common fractions; common fractions to decimals. Give the rule for the addition of decimals. Give the rule for the subtraction of decimals. Give the rule for the multiplication of decimals. Give the rule for the division of decimals.

UNITED STATES MONEY.

LESSON XXV.

ART. 99. The denominations of United States money, commonly used, are *dollars*, *cents*, and *mills*.

Accounts are kept in dollars and cents, mills being used only in making calculations. There are 10 mills in a cent, and 100 cents in a dollar.

ART. 100. The first two figures at the right of dollars denote cents, and the third figure denotes mills. The figures denoting dollars are preceded by the *dollar sign* ($), and they are separated from those denoting cents by the decimal point, called a *separatrix*. Thus, $45.307 denotes 45 *dollars* 30 *cents* 7 *mills*.

ART. 101. Cents and mills are decimal parts of a dollar, cents being *hundredths* and mills *thousandths*. $45.307 is a mixed decimal number, the dollars being expressed by integral orders; and the cents and mills, by decimal orders.

NOTE.—For the different kinds of money used, the coins minted, etc., see Art. 107.

WRITTEN EXERCISES.

Copy and read the following:

1.	2.	3.	4.
$3.45	$0.075	$40.045	$100.
$3.506	$0.005	$15.15	$405.
$1.055	$3.08	$10.015	$704.50
$0.75	$9.009	$60.60	$800.08

5. Write in figures 4 dollars 40 cents 3 mills.
6. Write 60 dollars 6 cents 4 mills.
7. Write 75 cents 5 mills; 40 cents 7 mills.

8. Write 300 dollars 3 cents 7 mills.
9. Write 500 dollars 5 cents; 500 dollars 5 mills.
10. Write 10 dollars 3 cents 8 mills.
11. Write 1000 dollars; 60 dollars.
12. Write 50 cents; 5 cents; 5 mills; 5 cents 5 mills.
13. How many cents in $15? *Ans.* 1500 cts.
14. Reduce to cents: $75; $108; $60; $73.
15. Reduce to cents: $125; $500; $150; $1000.
16. How many cents in $12.65? *Ans.* 1265 cts.
17. Reduce to cents: $6.40; $10.05; $20.04; $100.40.
18. How many mills in $25? *Ans.* 25000 mills.
19. Reduce to mills: $24; $46; $50; $100.
20. How many mills in $4.375? *Ans.* 4375 mills.
21. Reduce to mills: $3.125; $4.50; $6.12½; $4.37½.
22. Reduce to mills: $.625; $.62½; $4.16½; $.06
23. How many dollars in 7500 cents? *Ans.* $75.
24. Reduce to dollars: 1250 cents; 1805 cents.
25. How many dollars in 15000 mills? *Ans.* $15.
26. Reduce to dollars: 1500 mills; 10250 mills.
27. Reduce to dollars: 5000 mills; 500 mills.
28. Reduce to dollars: 375 cents; 1375 mills; 405 cents.
29. Reduce $4.50 to mills; $10.10 to cents.
30. Reduce 450 cents to dollars; 3500 mills to dollars.
31. Reduce $.37½ to mills; 1054 mills to dollars.

ART. 102. 1. To reduce dollars to cents:

Rule.—*Annex two ciphers.*

2. To reduce dollars to mills:

Rule.—*Annex three ciphers.*

3. To reduce cents to mills:

Rule.—*Annex one cipher.*

4. To reduce dollars and cents to cents; or dollars, cents, and mills to mills:

Rule.—*Remove the separatrix and the dollar sign.*

5. To reduce cents to dollars:

Rule.—*Place the separatrix before the second figure from the right, and prefix the dollar sign.*

6. To reduce mills to dollars:

Rule.—*Place the separatrix before the third figure from the right, and prefix the dollar sign.*

LESSON XXVI.

ADDITION AND SUBTRACTION.

WRITTEN PROBLEMS.

1. What is the sum of \$50, \$16.50, \$3.333, and \$.87½?

PROCESS.

\$50.
16.50
3.333
.875
\$70.708, *Ans.*

Write the several numbers to be added so that units of the same denomination and order stand in the same column, and then add, as in simple numbers.

The dollar sign need be written but twice,—before the first additive number and before the answer.

2. What is the sum of \$1.20, \$5, \$10.15, \$.85, \$.62½?

3. What is the sum of \$9, \$12.50, \$4.37½, \$40.08, \$6.33, \$8.75, \$13.12½, \$24.62½, and \$25.?

4. Add \$45.37½, \$100.50, \$16.12½, \$37, \$9.05, \$.87½, \$4.44, \$9.63, \$43.62½, \$1.08, \$40.87½, and \$95.

5. From \$37.50 take \$5.62½.

6. From \$6.37½ take \$5.87¾.

7. From \$100 take \$1.256.

8. From \$10 take \$.10

PROCESS. (5)

\$37.500
5.625
\$31.875, *Ans.*

9. A man sold a carriage for \$160.75, a horse for \$125, a set of harness for \$26.37½, and a saddle for \$15.62½: what was the amount received?

10. A grocer buys flour at \$8.62½ a barrel, and sells it at \$10 a barrel: what is his gain?

11. A merchant paid \$32.50 for a barrel of sugar, and sold it for \$35: how much did he gain?

12. A laborer earns \$17.50 a week, and his expenses are \$12.62½ a week: how much can he save each week?

13. A man bought a house and lot for \$3506.75, and sold it for \$4000: what was his gain?

14. A man bought a carriage for \$160, paid \$22.75 for repairing it, and then sold it for \$180: how much did he lose?

15. Mr. Smith bought a house and lot for \$4500, paid \$40.50 for a fence, \$105.65 for painting, \$47.12 for papering, and \$25 for other improvements: what will he make if he sell the property for \$5600?

ART. 103. To add or subtract sums of money:

Rule.—*Write units of the same denomination and order in the same column, add or subtract as in simple numbers, separate dollars and cents by a decimal point, and prefix the dollar sign.*

LESSON XXVII.

MULTIPLICATION AND DIVISION.

WRITTEN PROBLEMS.

1. What will 9 cords of wood cost at \$3.62½ a cord?

2. What will 16 barrels of flour cost at \$7.50 a barrel?

PROCESS.

\$3.625
9
\$32.625, *Ans.*

3. What will 40 yards of cloth cost at \$1.12½ a yard? At \$1.33⅓? At \$1.40?

4. What will 12⅗ tons of hay cost at \$11.50 a ton?

5. At 18¾ cents a dozen, what will 12 dozen eggs cost? 20 dozen? 36 dozen? 45 dozen?

6. If a boy earn \$4.37½ a week, how much will he earn in 16 weeks? In 32 weeks? 25 weeks?

7. A drover sold 36 cows at \$33.33⅓ a head: how much did he receive for them?

8. What will 90 bushels of wheat cost at \$1.62½ a bushel? At \$1.12½? At \$1.18½?

9. If 9 cords of wood cost \$32.62½, what will 1 cord cost?

PROCESS. (9)

9)\$32.625

\$3.625, *Ans.*

10. If 18 pounds of sugar cost \$2.16, what will 1 pound cost?

11. A man paid \$1687.50 for 45 acres of land: what was the price per acre?

12. A grocer paid \$135 for 18 barrels of flour: what was the cost a barrel?

13. A man earned \$91 in 8 weeks: how much did he earn a week?

14. At \$.12½ a dozen, how many dozens of eggs can be bought for \$5?

Reduce both divisor and dividend to the same denomination before dividing. \$.12½ = 125 mills, and \$5. = 5000 mills.

PROCESS.

125 m.)5000 m.(40, dozens.
500
0

15. At \$1.25 a bushel, how many bushels of corn can be bought for \$75?

16. At 3⅓ cents apiece, how many lemons can be bought for \$1.20? For \$3? For \$7?

17. If a boy earn 75 cents a day, in how many days will he earn \$6.75? \$13.50? \$24?

18. At 37½ cents a bushel, how many bushels of oats can be bought for \$15? For \$55.75?

19. Multiply \$12.62½ by 15, and divide the product by \$2.525

20. Multiply \$1.25 by 18, and divide the product by \$.62½.

21. Divide \$5.50 × 18 by \$.45

22. Multiply \$6.30 ÷ \$.09 by 12½.

ART. 104. 1. To multiply or divide sums of money:

Rule.—*Multiply or divide as in simple numbers, separate dollars and cents in the result by the decimal point, and prefix the dollar sign.*

2. To divide one sum of money by another:

Rule.—*Reduce both numbers to the same denomination, and divide as in simple numbers.*

LESSON XXVIII.

MISCELLANEOUS WRITTEN PROBLEMS.

1. Add \$13.45, \$9.87, \$100, \$.87, \$1.40, and \$14?
2. From \$10 take 5 mills; from \$500 take 500 cents.
3. Multiply \$15.33⅓ by 33.
4. Divide \$50 by 50 cents.
5. Multiply \$5.75 by 25, and divide product by \$.57½.
6. A man's income tax in 1868 was \$55.75, his state and city tax \$68.35, and his other taxes \$7.50: what was the amount of his taxes?
7. A man bought a house and lot for \$5400, and, after expending \$1500 for improvements, sold the property for \$7500: how much did he gain?
8. What will 60 pounds of butter cost at 30 cents a pound? At 27 cents? At 33⅓ cents?
9. What is the cost of 35 reams of paper, weighing 44 pounds each, at 18 cents a pound?
10. How many yards of carpeting, at \$1.75 a yard, can be bought for \$87.50? For \$350?
11. A fruit dealer makes a net profit of 20 cents on each bushel of apples he sells: how many bushels must he sell to make \$80?
12. A widow received one third of an estate of \$12000, and the remainder was divided equally between 5 children: what was the share of each child?

13. A fruit dealer sold 144 baskets of peaches for $252: what was the price per basket?

14. A farmer sold 35 pounds of butter at 20 cents a pound, and was paid in muslin at 12½ cents a yard: how many yards of muslin did he receive?

15. A man sold 15 cords of wood at $4.50 a cord, and received in payment 10 barrels of flour: what did the flour cost him a barrel?

16. If 8 barrels of salt cost $36, what will 13 barrels cost? 52 barrels? 64 barrels?

17. A grain dealer bought 15000 bushels of wheat at $1.35 a bushel, and sold it the next week for $1.48 a bushel: what was his gain?

18. A workman receives $1.50 a day, and his living costs him $.75 a day: how much can he lay up in a year, or 365 days, if he work 310 days?

19. A drover bought 240 sheep at $4.50 a head, drove them to market at an expense of $75, and then sold them at $6.50 a head: how much did he make?

20. A farmer exchanged 40 pounds of butter at 22 cents a pound, and 8 dozens of eggs at 12½ cents a dozen, for cotton cloth at 10 cents a yard: how many yards of cloth did he receive?

21. A farmer exchanged 8 cows, valued at $37.50 a head, for sheep valued at $7.50 a head: how many sheep did he receive?

22. A farmer sold 200 bushels of wheat, at $1.25 a bushel; 500 bushels of corn, at 78 cents a bushel; 65 bushels of potatoes, at 80 cents a bushel; 12 tons of hay, at $16.50 a ton; and 225 pounds of butter, at 20 cents a pound: what was the amount of his product?

23. What will be the cost of digging a cellar of 437.24 cubic yards at $.65 a cubic yard?

24. A bookseller sold 12 geographies, at $1.75; 20 readers, at $.85; 30 arithmetics, at $.65; and 45 spellers, at $.30: what was the amount of the bill?

25. A man bought 250 bushels of coal, at 15 cents a bushel; 7 cords of wood, at \$5.50 a cord; 18 bushels of potatoes, at \$.90 a bushel; and 9 barrels of apples, at \$2.75 a barrel: how much did he pay for all?

26. The annual expenses of a man's family are as follows: provisions, \$350; clothing, \$400; fuel, \$95; books and periodicals, \$50; house-rent, \$240; and all other expenses, \$150: if he receive an annual salary of \$1500, how much can he lay up each year?

27. If a boy pays \$2.50 a hundred for papers, and sells them at 5 cents apiece, how much does he make on 100 papers?

28. What is the cost of 8.75 cords of wood at \$4.50 a cord?

29. What is the cost of 5.4 tons of coal at \$7.25 a ton?

30. What is the cost of $12\frac{1}{2}$ yards of carpet at \$1.25 a yard?

N. B.—See "Manual of Arithmetic" for additional problems.

LESSON XXIX.

BILLS.

1. CINCINNATI, O., JAN. 10, 1882.

MR. CHARLES WILSON,

Bought of JAMES COOPER & Co.

13 lbs. Coffee,	@ 30c.	\$3.90
4 lbs. Butter,	@ 35c.	1.40
10 lbs. B'k't flour,	@ 6c.	.60
12 lbs. Dried beef,	@ 24c.	2.88
25 lbs. Sugar,	@ 18c.	4.50
3 lbs. Starch,	@ 20c.	.60
	Received payment,	\$

JAMES COOPER & Co.

What is the amount of the above bill?

Copy, fill, foot, and receipt each of the four following bills:

2. CHICAGO, JAN. 3, 1882.

JOSEPH MASON,

Bought of PETER & BROTHERS,

27	yds. Brussels carpeting,	@	$2.60	. . . . .
23	yds. Ingrain "	@	1.75	. . . . .
8¾	yds. Oil Cloth,	@	1.20	. . . . .
32	yds. Curtains,	@	.60	. . . . .
27	yds. Matting,	@	1.12½	. . . . .
16	Stair rods,	@	.22	. . . . .
4	Door mats,	@	1.40	. . . . .
2	Brussels rugs,	@	4.50	. . . . .

$

Received payment,

PETER & BROTHERS.

Per SMITH.

3. NASHVILLE, TENN., OCT. 8, 1882.

SAMUEL MILLS,

To JONES, SMITH & Co., *Dr.*

To 7	yds. Broadcloth,	@	$6.50	. . . . .
" 3½	yds. Doeskin,	@	2.75	. . . . .
" 7¾	yds. Linen,	@	.90	. . . . .
" 2½	doz. Handkerchiefs,	@	1.50	. . . . .
" 12½	yds. Muslin,	@	.18	. . . . .
" 9	yds. " bleached,	@	.33	. . . . .
" 12	yds. Silk,	@	1.60	. . . . .
" 19	yds. Binding,	@	.08	. . . . .
" 42	yds. Irish linen,	@	.62½	. . . . .
" 16½	yds. Alpaca,	@	.22	. . . . .
" 15	yds. Silk ribbon,	@	.37½	. . . . .

Received payment, $

4. St. Louis, Jan. 20, 1883.

Henry Williams,

Bought of Isaac Clarke.

1882.						
Mch.	10,	5	Pair	Calf Boots,	@	$5.75
"	"	8	"	Ladies' Gaiters,	@	3.10
"	"	7	"	Children's Shoes,	@	1.75
Apr.	4,	8	"	Coarse Boots,	@	2.75
"	"	6	"	Calf Shoes,	@	3.25
"	"	7	"	Ladies' Slippers,	@	1.20
May	33,	3	"	Calf Boots,	@	5.75

$

Received payment,

5. Indianapolis, Ind., Aug. 18, 1881.

Thos. M. Cochrane,

Bought of Jones, Dunlap & Co.

1881.						
June	20,	12	doz.	Scythes,	@	$15.
"	"	12½	"	Scy. Snaths,	@	16.50
"	"	6	"	Rakes,	@	2.25
"	"	8⅔	"	Whetstones,	@	1.50
Aug.	1,	5	"	Spades,	@	5.75

$

Cr.

June	20,	By Cash	$75.00
Aug.	1,	By 2½ doz. Scythes returned	37.50
			$112.50

Received payment, $

6. Andrew Wilson, bought of Smith & Waring, Indianapolis, Ind., Dec. 15, 1881, 7 gross shirt buttons, @ $4.50; 10 doz. linen napkins, @ $2.75; 8 doz. pairs kid gloves, @ $12.50; 3½ doz. linen handkerchiefs; and doz. pairs of socks, @ $4.50. Make out and receipt this bill, using Bill 1 (p. 178) as a model.

7. John Adams bought of Andrew Jackson, Cleveland, O., Feb. 10, 1882, 46 pounds of bacon, @ 15 cts.; 12½ pounds of dried beef, at 18 cts.; 62¼ pounds of lard, @ 11 cts.; 8 pounds of roast, at 11 cts.; 15 pounds of mutton chops, at 8 cts.; and 13 pounds of steak, at 12½ cts. Make out and receipt this bill as clerk of Andrew Jackson, using Bill 2 (p. 179) as a model.

8. Dr. J. W. Hamilton bought of Henry Wilson, Detroit, Mich., Jan. 5, 1882, 6½ tons of hard coal, @ $6.50; 4 tons of soft coal, @ $4.80; 5⅜ cords of wood, @ $5.75; 4½ cords of stove wood, @ $6.25; 15 bushels of charcoal, @ 22 cts.; 3 loads of kindling wood, @ $1.75; and 2 barrels of lime, @ $1.80. Jan. 14, Dr. Hamilton paid $20, and Feb. 4, $15. Make out and receipt this bill, using Bill 5 (p. 180) as a model.

9. Paul Jones bought of Charles Reeder, Fort Wayne, Ind., Feb. 1, 1882, 32 pounds of sugar, @ 12½ cts.; 14 pounds of coffee, @ 33⅓; 20 pounds of soap, @ 9 cts.; 10 pounds of rice, @ 11 cts.; 44 pounds of butter, @ 22 cts.; 20 pounds of lard, @ 9 cts.; 13 pounds of crackers, @ 9 cts.; 14 pounds of turkey, @ 12½ cts.; 3 chickens, @ 35 cts.; and 95 pounds of flour, @ 4½ cts. Make out and receipt this bill as clerk of Charles Reeder, using Bill 4 (p. 180) as a model.

10. Smith & Bell bought of Charles Aston, Columbus, O., Feb. 1, 1881, 2½ M. envelopes, @ $5.75; 1½ reams of cap paper, @ $8 per ream; and 3 blank books, @ $1.25 each; and, March 9, 1881, 5 dozen lead pencils (extra), @ $1.25 per dozen; 50 lbs. wrapping paper, @ 10 cts. per pound; and 6 vols. Dickens, @ $1.75 per vol. June 20, 1881, Mr. Aston received of Smith & Bell, by printing 1500 circulars, $5.50, and 1000 letter-heads, $3.75; and June 25, by 33 tokens press-work, at 50 cts. a token. Make out this bill with date of July 1, 1881, using Bill 5 (p. 180) as a model, and receipt it as clerk of Charles Aston.

DEFINITIONS.

ART. 105. A **Bill of Goods** is a written statement of goods sold, with the amount and price of each article and the entire cost. It also gives the date and place of the sale, and the names of buyer and seller.

A bill is drawn against the buyer, or *Debtor*, and in favor of the seller, or *Creditor*.

A bill is receipted by writing the words "*Received payment*" at the bottom, and affixing the seller's name. A bill may be receipted by a clerk, agent, or any other authorized person, as in Bill 2, page 179.

ART. 106. When sales are made at different times, the dates may be written at the left, as in Bills 4 and 5, page 180.

A bill presenting a debit and credit account between the parties, may be made out as in Bill 5.

QUESTIONS FOR REVIEW.

What are the denominations of United States money in common use? In what denominations are accounts kept? For what purpose are mills used? What use is made of the dollar sign? How are dollars and cents separated? Is the separatrix a period or a comma? Where is the figure denoting mills written?

How are dollars reduced to cents? Dollars to mills? Cents to mills? How are dollars and cents reduced to cents? Dollars, cents, and mills to mills? How are cents reduced to dollars? Mills to dollars?

Give the rule for adding or subtracting sums of money. Give the rule for multiplying or dividing a sum of money by an abstract number. Give the rule for dividing one sum of money by another.

What is a bill of goods? What does it contain? Against whom is it drawn? How is a bill receipted? By whom? Where are the dates of sales written? Items of credit?

DENOMINATE NUMBERS.

LESSON XXX.

UNITED STATES MONEY.*

ART. 107. **United States Money** is the legal currency of the United States. It is also called *Federal Money.*

The denominations are *mills*, *cents*, *dimes*, and *dollars.*

TABLE.

10 mills (*m.*)	. . are 1 cent	. . .	*c.* or *cts.*
10 cents	are 1 dime	. .	*d.*
10 dimes . . .	are 1 dollar	. .	$

$1 = 10*d.* = 100*c.* = 1000*m.*

NOTES.—1. United States money consists of *Coin* and *Paper Money.* Coin is called *Specie Currency*, or *Specie*, and paper money is called *Paper Currency.*

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*This lesson is the conclusion of the treatment of United States Money, and it is also made the introduction to the reduction of Denominate Numbers.

2. The principal gold coins are the double eagle ($20), eagle ($10), half-eagle ($5), quarter-eagle ($2½), three-dollar piece, and dollar.

The silver coins, now being coined, are the dollar, half-dollar, quarter-dollar, and dime.

The nickel coins (nickel and copper) are the five-cent piece and three-cent piece. The cent and two-cent piece are made of bronze, an alloy of copper, tin, and zinc.

3. Paper money consists of notes issued by the United States, called *Treasury Notes*, and bank notes issued by National Banks.

ORAL EXERCISES.

1. How many mills in 1 cent? In 5 cents? 4 cents? 7 cents? 9 cents? 10 cents?

2. How many cents in 1 dime? In 4 dimes? 5 dimes? 8 dimes? 10 dimes? 15 dimes?

3. How many dimes in 1 dollar? In 3 dollars? 6 dollars? 4 dollars? 8 dollars? 10 dollars?

4. How many cents in 10 mills? In 50 mills? 40 mills? 60 mills? 80 mills? 100 mills?

5. How many dimes in 40 cents? In 90 cents? 60 cents? 80 cents? 70 cents? 100 cents?

6. How many dollars in 50 dimes? In 70 dimes? 60 dimes? 90 dimes? 80 dimes? 100 dimes?

7. How many dimes in 25 cents? In 75 cents? 15 cents? 65 cents? 35 cents? 95 cents?

8. How many cents in 35 mills? In 65 mills? 25 mills? 85 mills? 75 mills? 95 mills?

9. How many cents in 5 dimes? In 15 dimes? 25 dimes? 30 dimes? 40 dimes? 50 dimes?

10. How many dimes in 15 cents? In 95 cents? 85 cents? 65 cents? 80 cents? 75 cents?

11. How many cents in 1 dollar? In 5 dollars? 7 dollars? 9 dollars? 8 dollars? 10 dollars?

12. How many dollars in 200 cents? In 500 cents? 400 cents? 900 cents? 700 cents? 1000 cents?

LESSON XXXI.

DRY MEASURE.

ART. 108. **Dry Measure** is used in measuring grain, seeds, fruit, vegetables, coal, and many other dry articles.

The denominations are *pints*, *quarts*, *pecks*, and *bushels*.

TABLE.

2 pints (*pt.*)	are 1 quart		*qt.*
8 quarts	are 1 peck		*pk.*
4 pecks	are 1 bushel		*bu.*

1 bu. = 4 pk. = 32 qt. = 64 pt.

NOTES.—1. The standard bushel is 18½ inches in diameter and 8 inches deep. It contains 2150⅖ cubic inches.

2. In measuring grain, seeds, and small fruits, the measure must be *even* full; but in measuring corn in the ear, potatoes, apples, and other large articles, the measure must be *heaping* full.

ORAL EXERCISES.

1. How many pints in 3 quarts?

SOLUTION.—In 3 quarts there are 3 times 2 pints, which is 6 pints.

2. How many pints in 5 quarts? In 8 quarts?

3. How many quarts in 10 pints?

SOLUTION.—In 10 pints there are as many quarts as 2 pints are contained times in 10 pints, which is 5 times. There are 5 quarts in 10 pints.

4. How many quarts in 8 pints? 14 pints? 16 pints? 20 pints? 18 pints? 24 pints?

5. How many quarts in 3 pecks? 5 pecks? 7 pecks? $4\frac{1}{2}$ pecks? $6\frac{1}{4}$ pecks? $10\frac{3}{4}$ pecks?

6. How many pecks in 16 quarts? 20 quarts? 32 quarts? 56 quarts? 48 quarts? 64 quarts?

7. How many pecks in 5 bushels? 6 bushels? 10 bushels? $7\frac{1}{2}$ bushels? $9\frac{3}{4}$ bushels? 11 bushels?

8. How many bushels in 12 pecks? 20 pecks? 32 pecks? 40 pecks? 36 pecks? 44 pecks?

9. How many quarts in 8 pecks? In 12 pints?

10. How many pecks in 24 quarts? In 5 bushels?

11. What part of a quart is 1 pint? 3 pints?

12. What part of a peck is 1 quart? 3 quarts?

13. What part of a bushel is 1 peck? 2 pecks? 3 pecks? 5 pecks?

14. What part of a peck is 3 pints? 5 pts.? 8 pts.?

15. How many pecks in 17 quarts? In 27 quarts? In 33 quarts? 40 qts.? 56 qts.?

16. How many bushels in 13 pecks? In 23 pecks? In 33 pecks? 27 pks.? 42 pks.? 50 pks.?

17. How many pecks in 5 bushels? How many quarts? How many pints?

WRITTEN EXERCISES.

18. Reduce 12 bushels to pints.

19. Reduce 12 bu. 3 pk. 1 pt. to pints.

12 bu. = 12 times 4 pk. = 48 pk., and 48 pk. + 3 pk. = 51 pk.

51 pk. = 51 times 8 qt. = 408 qt.; and 408 qt. = 408 times 2 pt. = 816 pt., and 816 pt. + 1 pt. = 817 pt.

PROCESS.

bu. pk. qt. pt.

12 + 3 + 0 + 1

4

51, pk.

8

408, qt.

2

817, pt., *Ans.*

20. Reduce 15 bu. 1 pk. 5 qt. 1 pt. to pints.

21. Reduce 6 bu. 2 pk. 1 pt. to pints.

22. Reduce 3 pk. 7 qt. 1 pt. to pints.

23. Reduce 5 bu. 2 pk. 7 qt. to pints.

24. Reduce 15 bu. 5 qt. 1 pt. to pints.

25. Reduce 8 bu. 3 pk. to quarts.

26. Reduce 3 pk. 7 qt. to quarts.

27. Reduce 1 bu. 5 qt. to quarts.

28. Reduce 11 bu. 1 pt. to pints.

29. How many bushels in 768 pints? In 817 pints?

PROCESS.

2)817, pt.

8)408, qt. + 1 pt.

4) 51, pk.

12, bu. + 3 pk.

Ans. 12 bu. 3 pk. 1 pt.

There are as many qts. in 817 pts. as there are times 2 pts., which is 408, with 1 pt. remainder; there are as many pks. in 408 qts. as there are times 8 qts., which is 51; there are as many bushels in 51 pks. as there are times 4 pks., which is 12, with 3 pks. remainder. Hence, 817 pt. = 12 bu. 3 pk. 1 pt.

NOTE.—The divisors may be regarded as abstract numbers—2, 8, and 4 respectively. In 817 pts. there are $\frac{1}{2}$ as many qts. as pts., $\frac{1}{8}$ as many pks. as qts., and $\frac{1}{4}$ as many bus. as pks.

30. Reduce 68 qt. to bushels; 168 qt. to bushels.

31. Reduce 42 pt. to bushels; 342 pt. to bushels.

32. Reduce 51 pt. to pecks; 90 pt. to pecks.

33. How many pecks in 37 pints? 45 pints?

34. How many bushels in 151 quarts? 125 quarts?

35. Reduce 17 bu. to qt.; 172 qt. to bushels.

36. Reduce 3 pk. 5 qt. to pt.; 201 pt. to pecks.

37. Reduce 21 bu. 1 qt. to qt.; 903 pt. to bushels.

38. A man sold 1 bu. 3 pk. 5 qt. of clover-seed at 8 cents a quart: what did he receive?

39. A fruit dealer paid $7 for 3 bu. 3 pk. of peaches, and sold them at 75 cents a peck: what was his gain?

40. How many bushels of chestnuts can be bought for $15.50, at 5 cents a quart? At 10 cts. a quart?

41. A boy bought 5 pecks of cherries at 60 cents a peck, and sold them at 10 cents a quart: how much did he gain?

To Teachers. —See "Manual of Arithmetic" for additional problems in Denominate Numbers.

LESSON XXXII.

LIQUID MEASURE.

Art. 109. **Liquid Measure** is used in measuring liquids; as, oil, milk, alcohol, etc.

The denominations are *gills*, *pints*, *quarts*, and *gallons*.

TABLE.

4 gills (*gi.*) . .	are 1 pint . . .	*pt.*
2 pints . . .	are 1 quart . . .	*qt.*
4 quarts . . .	are 1 gallon . .	*gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

Notes.—1. The standard liquid gallon contains 231 cubic inches.

2. The size of casks for liquids is variable. Barrels generally contain 31½ gallons, and hogsheads 63 gallons.

ORAL EXERCISES.

1. How many gills in 3 pints? In 10 pints? 20 pints? 32 pints? 40 pints? 24 pints?

2. How many pints in 5 quarts? 8 quarts? 12 quarts? $10\frac{1}{2}$ quarts? $12\frac{1}{2}$ quarts? 20 quarts?

3. How many quarts in 5 gallons? 6 gallons? 8 gallons? $7\frac{3}{4}$ gallons? 11 gallons? $12\frac{1}{2}$ gallons?

4. How many pints in 16 gills? 24 gills? 32 gills? 36 gills? 40 gills? 42 gills? 48 gills?

5. How many quarts in 12 pints? 16 pints? 22 pints? 32 pints? 40 pints? 48 pints?

6. How many gallons in 20 quarts? 32 quarts? 28 quarts? 36 quarts? 40 quarts? 48 quarts?

7. How many quarts in 8 pints? 15 pints? 19 pints? 13 pints? 21 pints? 17 pints? 25 pints?

8. How many pints in 8 quarts? 11 quarts? 16 quarts? $15\frac{1}{2}$ quarts? 20 quarts? $20\frac{1}{2}$ quarts?

9. How many gallons in 8 quarts? 13 quarts? 21 quarts? 24 quarts? 29 quarts? 32 quarts?

10. How many quarts in 6 gallons? $9\frac{3}{4}$ gallons? $10\frac{1}{2}$ gallons? $12\frac{1}{2}$ gallons? $15\frac{1}{4}$ gallons?

11. What part of a gallon is 1 quart? 2 quarts? 3 quarts? 5 quarts? 7 quarts?

12. What part of a pint is 2 gills? 3 gills? 5 gills?

13. How many quarts in $\frac{3}{4}$ of a gallon? $\frac{5}{8}$ gallon?

14. How many pints in $5\frac{1}{2}$ gallons? $10\frac{1}{4}$ gallons?

WRITTEN EXERCISES.

15. Reduce 21 gallons to pints; gills.

16. Reduce 13 gal. 1 qt. 1 pt. 3 gi. to gills.

17. Reduce 24 gal. 3 qt. 1 gi. to gills.

18. Reduce 23 gal. 1 pt. to pints.

19. Reduce 7 gal. 3 qt. 3 gi. to gills.

20. Reduce 9 gal. 2 qt. 1 pt. to pints.

21. Reduce 38 pt. to gallons; 75 pt. to quarts.

22. Reduce 45 gi. to gallons; 245 gi. to gallons.

23. Reduce 97 gi. to quarts; 130 gi. to quarts.

24. Reduce $25\frac{1}{2}$ gal. to gi.; $45\frac{1}{2}$ gal. to gills.

25. Reduce 56 gal. 1 pt. to pints; 137 pt. to gallons.

26. Reduce 305 pt. to gallons; 43 gal. 3 qt. to pints.

27. Reduce 420 pt. to gallons; 62 qt. to gills.

28. What will 3 gal. 5 qt. of plums cost at 8 cents a quart? At 10 cts. a quart?

29. What will 256 pints of maple syrup cost at $1.30 a gallon? At $1.10 a gallon? At 90 cts. a gallon?

30. How many vials, holding 2 gills each, can be filled from $5\frac{1}{4}$ gallons of alcohol?

LESSON XXXIII.

LONG MEASURE.

ART. 110. **Long Measure** is used in measuring lines or distances. It is also called *Linear Measure.*

The denominations in common use are *inches, feet, yards, rods,* and *miles.*

TABLE.

12	inches (*in.*) . .	are 1 foot . . .	*ft.*
3	feet	are 1 yard . . .	*yd.*
5½	yards or 16½ feet	are 1 rod . . .	*rd.*
320	rods	are 1 mile . . .	*mi.*

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

NOTE.—The mile is commonly divided into halves, fourths, eighths, etc. The term furlong (⅛ of a mile) is now seldom used.

The following denominations are also used:

4 in. = 1 hand; used in measuring the height of horses.
3 ft. = 1 pace.
6 ft. = 1 fathom; used in measuring the depth of water.
3 mi. = 1 league; used in measuring distances at sea.
60 geographic miles = 1 degree at the equator.
69½ statute miles (nearly) = 1 degree at the equator.
360 degrees (°) make the circumference of the earth.

NOTE.—In measuring cloth, ribbons, etc., the width is not considered, and the yard is divided into *halves, fourths, eighths,* etc.

ORAL EXERCISES.

1. How many inches in 1 foot? 5 feet? 8 feet?
2. How many feet in 36 inches? In 72 in.? 84 in.?
3. How many feet in 4 yards? In 9 yards? 12 yd.?
4. How many yards in 15 feet? 21 feet? 36 ft.?
5. How many yards in 2 rods? 6 rods? 10 rd.?
6. How many yards in 5 rods? 7 rods? 9 rd.?
7. How many fathoms in 36 feet? 60 feet?
8. How many feet in 2 rods? 10 rd.?
9. How many rods in 2 miles? 10 mi.?
10. What part of a foot is 9 inches? 7 in.? 8 in.? 10 in.? 11 in.? 5 in.? 6 in.?
11. What part of a yard is 2 feet? 5 ft.? 7 ft.?
12. A horse is 15 hands high: what is its height in feet?

13. A vessel sank in water 9 fathoms deep: what was the depth of water in feet?

14. A steamer was wrecked 5 leagues from shore: how many miles?

15. How many yards of ribbon in a piece 40 feet long?

WRITTEN EXERCISES.

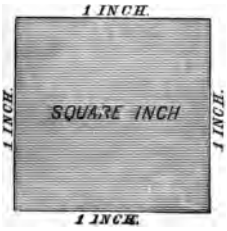
16. Reduce 140 rd. 3 yd. to inches.
17. Reduce 110 rd. 4 ft. to feet.
18. Reduce 3 mi. 220 rd. to yards.
19. Reduce 5 mi. 20 rd. 2 ft. to inches.
20. Reduce 17 mi. 150 rd. to rods.
21. Reduce 4 mi. 27 rd. 2 ft. 10 in. to inches.
22. Reduce 500 rd. to miles; 3520 rd. to miles.
23. Reduce 4124 yd. to miles; 12372 yd. to miles.
24. How many feet in $2\frac{1}{2}$ miles?
25. How many inches in 16.4 rods?
26. How many miles in 15840 feet?

LESSON XXXIV.

SQUARE MEASURE.

ART. 111. **Square Measure** is used in measuring surfaces. It is also called *Superficial Measure* and *Land Measure.*

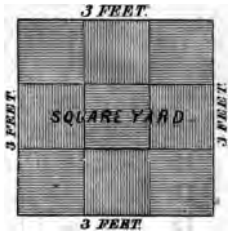
The denominations are *square inches*, *square feet*, *square yards*, *square rods* (or *perches*), *acres*, and *square miles.*



ART. 112. A **Square Inch** is a square, each side of which is an inch in length.

The figure at the left represents a square inch of real size.

A **Square Yard** is a square, each side of which is a yard, or three feet, in length. It contains 9 square feet.



NOTE.—The teacher should explain and illustrate a right angle, a rectangle, and a square.

TABLE.

144	square inches (*sq. in.*)	are 1 square foot	*sq. ft.*
9	square feet	are 1 square yard	*sq. yd.*
$30\frac{1}{4}$	square yards . . .	are 1 square rod (or perch)	*sq. rd.*
160	square rods	are 1 acre	*A.*
640	acres	are 1 square mile	*sq. mi.*

NOTE.—Land surveyors use Gunter's Chain, which is 4 rods, or 66 feet, long, and consists of 100 links, each link being $7\frac{92}{100}$ inches long. A square chain is 16 square rods, and 10 square chains are 1 acre.

ORAL EXERCISES.

1. How many square feet in 3 square yards? In 5 sq. yd.? 7 sq. yd.? 9 sq. yd.? 10 sq. yd.?

2. How many square yards in 27 sq. ft.? 45 sq. ft.? 6 sq. ft.? 81 sq. ft.? 90 sq. ft.? 108 sq. ft.?

3. How many square inches in $\frac{1}{4}$ of a square foot? $\frac{1}{8}$ of a square foot? $\frac{1}{12}$ sq. ft.? $\frac{1}{24}$ sq. ft.? $\frac{1}{18}$ sq. ft.?

4. How many square feet in $\frac{1}{3}$ of a square yard? $\frac{2}{3}$ of a square yard? $\frac{1}{2}$ sq. yd.? $\frac{2}{9}$ sq. yd.? $\frac{3}{4}$ sq. yd.?

5. How many square rods in $\frac{1}{2}$ of an acre? $\frac{1}{4}$ of an acre? $\frac{3}{4}$ of an acre? $\frac{3}{8}$ of an acre?

6. How many square feet in $2\frac{1}{3}$ square yards? $2\frac{2}{3}$ square yards? $3\frac{2}{3}$ sq. yd.? $5\frac{1}{3}$ sq. yd.?

7. How many acres in $\frac{1}{2}$ of a square mile? $\frac{1}{4}$ sq. mi.? $\frac{1}{8}$ sq. mi.? $\frac{3}{8}$ sq. mi.? $\frac{3}{16}$ sq. mi.?

WRITTEN EXERCISES.

8. How many square rods in 12 acres? 25 A.?

9. Reduce 120 sq. rd. to square feet.

10. Reduce 40 A. 90 sq. rd. to square yards.

11. Reduce 4 sq. yd. 8 sq. ft. 32 sq. in. to square inches.

12. Reduce 10 A. 100 sq. rd. 10 sq. yd. 1 sq. ft. to square feet.

13. Reduce 5280 sq. rd. to acres.

14. Reduce 5184 sq. in. to square yards.

15. Reduce 584 sq. ft. to square yards.

16. Reduce 1728 sq. in. to square feet.

17. Reduce 70882 sq. ft. to higher denominations.

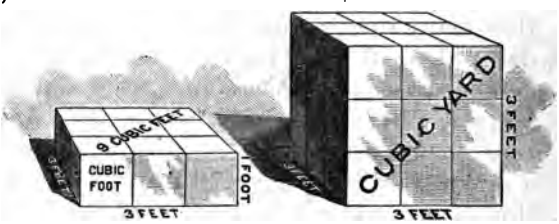
18. How many acres in $5\frac{1}{4}$ sq. mi.?

LESSON XXXV.

CUBIC MEASURE.

ART. 113. **Cubic Measure** is used in measuring solids. It is also called *Solid Measure.*

The denominations are *cubic inches, cubic feet, cubic yards,* and *cords.*



ART. 114. A **Cubic Inch** is a cube whose edges are each one inch long.

A **Cubic Foot** is a cube whose edges are each one foot long.

A **Cubic Yard** is a cube whose edges are each one yard long.

NOTE.—The teacher should explain a cube; also its faces and edges.

TABLE.

1728 cubic inches (*cu. in.*)	are 1 cubic foot . .	*cu. ft.*
27 cubic feet	are 1 cubic yard . .	*cu. yd.*
128 cubic feet	are 1 cord	*cd.*

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

WRITTEN EXERCISES.

1. How many cubic inches in 10 cubic feet?
2. How many cubic inches in $24\frac{1}{2}$ cubic feet?
3. How many cubic inches in 32 cubic feet?
4. How many cubic inches in $45\frac{1}{2}$ cubic feet?
5. How many cubic feet in 85 cubic yards?
6. How many cubic feet in $96\frac{1}{3}$ cubic yards?
7. How many cubic feet in 15 cords? $12\frac{3}{4}$ cd.?
8. How many cubic feet in 15552 cubic inches?
9. How many cubic feet in 120 cubic yards?
10. How many cubic yards in 351 cubic feet?
11. Reduce 405 cubic feet to cubic yards.
12. Reduce 15 cu. yd. 16 cu. ft. 1305 cu. in. to cubic inches.
13. Reduce 1536 cubic feet to cords.
14. Reduce 1473462 cu. in. to cubic yards.

LESSON XXXVI.

TIME MEASURE.

ART. 115. **Time Measure** is used in measuring time or duration.

The denominations are *seconds*, *minutes*, *hours*, *days*, *years*, and *centuries*.

TABLE.

60 seconds (*sec.*)	are 1 minute . . .	*min.*
60 minutes . .	are 1 hour	*h.*
24 hours . . .	are 1 day	*d.*
365 days . . .	are 1 common year .	*c. yr.*
366 days . . .	are 1 leap year . .	*l. yr.*
100 years (365¼ d.)	are 1 century . . .	*C.*

1 d. = 24 h. = 1440 min. = 86400 sec.

The following denominations are also used:

7 days	are 1 week	*w.*
4 weeks	are 1 lunar month .	*lr. m.*
365¼ days	are 1 Julian year .	*J. yr.*
12 calendar months	are 1 civil year . .	*c. yr.*

NOTES.—1. The exact length of a solar year is 365 d. 5 h. 48 min. 46 sec., which is nearly 6 hours, or ¼ of a day, longer than the common year. Since the common year lacks nearly ¼ of a day of the true time, an additional day is added to every fourth year, making *leap year*. This additional day is given to February, and hence this month in leap year contains 29 days. The leap years are exactly divisible by 4; as, 1880, 1884, etc. A year ending in two or more ciphers, as 1900, 2000, must be divisible by 400 in order to be a leap year.

2. The names and order of the calendar months and the number of days in each are as follows:

January,	1st	month,	31 days.	July,	7th	month,	31 days.
February,	2d	"	28 or 29.	August,	8th	"	31 "
March,	3d	"	31 days.	September,	9th	"	30 "
April,	4th,	"	30 "	October,	10th,	"	31 "
May,	5th,	"	31 "	November,	11th,	"	30 "
June,	6th,	"	30 "	December,	12th,	"	31 "

3. The following couplet will assist in remembering the months which have 30 days each:

Thirty days hath September,
April, June, and November.

4. In most business transactions 30 days are considered a month, and 360 days a year.

5. The year is divided into four seasons of three months each, as follows:

SPRING,	March, April, May,	AUTUMN or FALL,	September, October, November,
SUMMER,	June, July, August,	WINTER,	December, January, February,

ORAL EXERCISES.

1. How many seconds in 2 minutes? In 5 min.?
2. How many minutes in 3 hours? 5 hr.? 10 hr.?
3. How many hours in 120 minutes? In 240 min.? 300 min.? 600 min.?
4. How many hours in 2 days? 3 days? 5 days?
5. How many days in 48 hours? 72 hours? 240 hr.?
6. How many days in 6 weeks? 8 weeks? 5 w.? 9 w.? 10 w.? 12 w.? 11 w.?
7. How many weeks in 35 days? 49 days? 56 d.? 77 d.? 84 d.? 70 d.? 84 d.? 140 d.?
8. How many weeks in 5 lunar months? 7 lunar months? 8 lr. m.? 10 lr. m.? 12 lr. m.?
9. How many lunar months in 16 weeks? 32 weeks? 44 w.? 60 w.? 48 w.? 36 w.? 40 w.?
10. How many calendar months in 5 years? 7 years? 10 yr.? 12 yr.? 9 yr.? 8 yr.? 6 yr.?

WRITTEN EXERCISES.

11. How many seconds in 15 hours?
12. How many hours in 28800 seconds?
13. Reduce 5 d. 13 h. 40 min. to seconds.
14. Reduce 31 d. 30 min. 45 sec. to seconds.
15. Reduce 30600 minutes to higher denominations.
16. Reduce 52560 hours to common years.
17. How many minutes in a leap year?

18. How many seconds in a solar year, which contains 365 d. 5 h. 48 min. 46 sec.?

19. How many seconds in a common year?

20. The age of a certain man is 64 yr. 45 d.: how many days has he lived, allowing 365¼ days to the year? How many hours?

21. How many hours in the three spring months? In the three summer months? The three autumn months? The three winter months?

22. How many minutes will there be in the month of February, 1884? In February, 1886?

23. If your pulse beat 75 times a minute, how many times will it beat in 1 week? In 5 weeks?

24. How many days will it take a steamship to sail 3744 miles, if it sail at the rate of 12 miles an hour? How many weeks?

LESSON XXXVII.

AVOIRDUPOIS WEIGHT.

ART. 116. **Avoirdupois Weight** is used in weighing all articles except gold, silver, and precious stones.

The denominations are *ounces*, *pounds*, *hundred-weights*, and *tons*.

TABLE.

16 ounces (*oz.*) . .	are 1 pound	*lb.*
100 pounds	are 1 hundred-weight .	*cwt.*
20 hundred-weight	are 1 ton	*T.*

1 T. = 20 cwt. = 2000 lb. = 32000 oz.

NOTES.—1. The dram ($\frac{1}{16}$ of an ounce) is seldom used, and hence is not included in the table.

2. At the United States custom house, and in weighing coal and iron at the mines, the ton used contains 2240 pounds. It is called the *long* or *gross* ton.

3. In most of the States, 56 lb. corn or rye, 60 lb. wheat or potatoes, and 32 lb. oats are a bushel. A barrel of flour contains 196 lb., and a barrel of beef or pork 200 lb.

ORAL EXERCISES.

1. How many ounces in 2 pounds? 3 lb.? 4 lb.?
2. How many pounds in 32 ounces? 48 oz.? 96 oz.?
3. How many ounces in $2\frac{1}{2}$ pounds? $5\frac{1}{2}$ lb.?
4. How many pounds in 3 hundred-weight? 5 cwt.?
5. How many pounds in $2\frac{1}{2}$ cwt.? $5\frac{1}{4}$ cwt.? $6\frac{3}{4}$ cwt.?
6. How many cwt. in 3 tons? 4 tons? 5 tons?
7. How many ounces in $\frac{3}{4}$ of a pound? $\frac{5}{8}$ lb.? $\frac{7}{8}$ lb.?
8. How many cwt. in $\frac{1}{4}$ of a ton? $\frac{3}{5}$ T.? $\frac{3}{4}$ T.?

WRITTEN EXERCISES.

9. Reduce 5 tons to ounces.
10. Reduce 3 T. 14 cwt. 56 lb. to pounds.
11. Reduce 5 cwt. 77 lb. 13 oz. to ounces.
12. Reduce 34920 pounds to tons.
13. Reduce 4560 ounces to higher denominations.
14. Reduce 11 T. 38 lb. to ounces.
15. What will a barrel of flour cost at 6 cents a pound? $\frac{3}{4}$ of a barrel?

16. What will 3 barrels of pork cost at 15 cents a pound? At 12 cents a pound?

17. What will it cost to transport 50 T. 15 cwt. 75 lb. of freight, at $\frac{1}{2}$ cent a pound?

LESSON XXXVIII.

OTHER MEASURES OF WEIGHT.

To TEACHERS.—Since Troy Weight is used chiefly by jewelers, and Apothecaries Weight by physicians and druggists, it is recommended that their study be deferred until they are reached in the "New Complete Arithmetic." The tables are, however, here inserted for use if this be thought best. The teacher can readily dictate both oral and written exercises.

ART. 117. **Troy Weight** is used in weighing gold, silver, and precious stones.

The denominations are *grains*, *pennyweights*, *ounces*, and *pounds*.

TABLE.

24 grains (*gr.*)	are 1 pennyweight	.	*pwt.*
20 pennyweights	are 1 ounce	. . .	*oz.*
12 ounces	are 1 pound	. . .	*lb.*

ART. 118. **Apothecaries Weight** is used by physicians in prescribing, and by apothecaries in mixing or compounding medicines.

The denominations are *grains*, *scruples*, *drams*, *ounces*, and *pounds*.

TABLE.

20 grains (*gr.*)	are 1 scruple	. . ℈
3 scruples	are 1 dram	. . . ʒ
8 drams	are 1 ounce	. . . ℥
12 ounces	are 1 pound	. . . *lb.*

NOTE.—Medicines are bought and sold in quantities by avoirdupois weight.

LESSON XXXIX.

MISCELLANEOUS TABLE.

ART. 119. The following denominations are in common use:

PAPER.		NUMBERS.	
24 sheets	are 1 quire.	12 things	are 1 dozen.
20 quires	are 1 ream.	12 dozen	are 1 gross.
2 reams	are 1 bundle.	12 gross	are 1 great gross.
5 bundles	are 1 bale.	20 things	are 1 score.

A sheet of paper folded in 2 leaves is called a *folio;* in 4 leaves, a *quarto*, or *4to;* in 8 leaves, an *octavo*, or *8vo;* in 12 leaves, a *duodecimo*, or *12mo;* in 18 leaves, an *18mo.*

ORAL EXERCISES.

1. How many sheets of paper in $5\frac{1}{2}$ quires?
2. How many quires of paper in 4 reams? In 8 reams? $12\frac{3}{5}$ reams? 15 reams?
3. How many bundles of paper in 6 reams? In 12 reams? 18 reams? 32 reams?
4. How many eggs in 5 dozen? In $7\frac{3}{4}$ dozen? $8\frac{1}{3}$ dozen? 12 dozen? 20 dozen?
5. How many years are 3 score years? 3 score years and 10? 4 score and 10 years?

WRITTEN EXERCISES.

6. How many sheets of paper in $12\frac{1}{2}$ reams?
7. Reduce 6 reams 15 quires 12 sheets to sheets.
8. How many quires in 7200 sheets of paper? How many reams?
9. How many crayons are there in 36 boxes, if each box contains 1 gross?

10. If a shirt require 6 buttons, how many shirts will 12 gross of buttons trim? 20 gross?

11. What will 44 gross of lead-pencils cost at 75 cents a dozen? At 45 cts.? At $33\frac{1}{3}$ cts.?

12. A stationer bought 15 reams of letter-paper at $3.50 a ream, and sold it at 25 cents a quire: how much did he gain?

DEFINITIONS AND RULES.

ART. 120. A **Denominate Number** is a concrete number composed of one or several denominations; as, 3 feet; 12 oz.; 4 bu. 3 pk. 5 qt.

NOTE.—Every denominate number is concrete, but all concrete numbers are not denominate. 15 feet is both concrete and denominate; 15 oranges is concrete, but not denominate (Art. 27).

ART. 121. When a denominate number is composed of units of one denomination, as 12 ft., it is **Simple**; and when composed of units of *several* denominations, as 5 yd. 3 ft. 7 in., it is **Compound.**

A compound denominate number may properly be called a **Compound Number**, since every compound number is denominate.

ART. 122. The numbers expressing the successive denominations of a compound number are its *Terms.*

Compound numbers are of the *same kind* when their corresponding terms are of the same denomination; as, 3 bu. 2 pk. and 5 bu. 3 pk. 5 qt.

ART. 123. Denominate numbers express **Currency, Measures,** and **Weights.**

Currency includes United States Money, composed of coin and paper money.

The Measures include measures (1) of *lines*, Long Measure; (2) of *surfaces*, Square Measure; (3) of *solid contents*, Cubic Measure; (4) of *capacity*, Dry Measure and Liquid Measure; (5) of *duration*, Time Measure.

The Weights include Avoirdupois Weight, Troy Weight, and Apothecaries Weight.

NOTE.—For the Metric System of Weights and Measures, see "New Complete Arithmetic."

ART. 124. **Reduction** is the process of changing a denominate number from one denomination to another without altering its value.

The changing of a denominate number to a lower denomination is called *Reduction Descending*, and the changing of it to a higher denomination is called *Reduction Ascending.*

ART. 125. 1. To reduce a denominate number to a lower denomination:

Rule.—1. *Multiply the number of the highest denomination by the number of units of the next lower which equals a unit of the higher, and to the product add the number of the lower denomination, if any.*

2. *Proceed in like manner with this and each successive result thus obtained until the number is reduced to the required denomination.*

NOTE.—If the denominate number is compound, write the successive denominations in their proper order, filling vacant denominations, if any, with ciphers.

2. To reduce a denominate number to a higher denomination:

Rule.—1. *Divide the given denominate number by the number of units of its denomination which equals one unit of the next higher denomination, and place the remainder, if any, at the right.*

2. *Proceed in like manner with this and each successive quotient thus obtained until the number is reduced to the required denomination.*

3. *The last quotient, with the several remainders annexed in proper order, will be the answer required.*

LESSON XL.

ADDITION OF COMPOUND NUMBERS.

WRITTEN EXERCISES.

1. What is the sum of 5 bu. 3 pk. 6 qt.; 8 bu. 2 qt.; 10 bu. 3 pk. 3 qt.; and 3 pk. 4 qt.?

PROCESS.

bu.	pk.	qt.
5	3	6
8	0	2
10	3	3
	3	4
25 bu.	2 pk.	7 qt.

Write the terms of the same denomination in the same column. Add first the column of quarts; the sum is 15 quarts, which is 1 pk. 7 qt. Write the 7 qt. under the quarts, and add the 1 pk. with the column of pecks. The sum of the pecks is 10 pecks, which is 2 bu. 2 pk. Write the 2 pk. under the pecks, and add the 2 bu. with the column of bushels. The sum of the bushels is 25 bushels. The sum of the four compound numbers added is 25 bu. 2 pk. 7 qt.

NOTE.—In both simple addition and compound addition, the sum of the numbers of each denomination (or order) is divided *by the number of units of this denomination which equals one of the next higher.* In simple addition this divisor is 10, the scale of orders being uniform; in compound addition this divisor varies, the scale of the denominations being *irregular.*

	2.				**3.**				**4.**				
	bu.	pk.	qt.	pt.	gal.	qt.	pt.	gi.	mi.	rd.	yd.	ft.	in.
	16	2	6	1	21	3	1	3	19	319	5	2	10
	23	1	4	0	16	0	1	2	27	144	3	1	6
	40	3	0	1	48	2	0	0	45	143	0	0	7
Add	9	0	2	0	35	0	1	3	6	17	2	1	0

5. What is the sum of 15 w. 5 d. 22 h. 45 min. 34 sec.; 8 w. 6 d. 13 h.; 3 w. 20 h. 52 min.; 4 d. 22 h. 33 min. 55 sec.; 1 w. 2 d. 3 h. 30 min.?

6. Add 625 A. 155 sq. rd. 16 sq. yd.; 546 A. 108 sq. rd. 6 sq. yd.; 675 A. 124 sq. rd. 7 sq. yd.; 300 A. 156 sq. rd. $1\frac{1}{4}$ sq. yd.

7. The four quarters of an ox weighed respectively 2 cwt. 84 lb. 10 oz.; 3 cwt. 1 lb. 14 oz.; 2 cwt. 76 lb. 4 oz.; and 2 cwt. 98 lb. 14 oz.: what was the weight of the ox?

8. A garden has four unequal sides; the first is 4 rd. 3 yd. 2 ft. 8 in.; the second, 5 rd. 1 ft. 10 in.; the third, 4 rd. 5 yd. 4 in.; and the fourth, 3 rd. 4 yd. 2 ft. 9 in.: what is the distance round the garden?

ART. 126. **Compound Addition** is the process of finding the sum of two or more compound numbers of the same kind.

LESSON XLI.

SUBTRACTION OF COMPOUND NUMBERS.

WRITTEN EXERCISES.

1. From 35 rd. 5 yd. 1 ft. take 27 rd. 3 yd. 2 ft.

PROCESS.

rd.	yd.	ft.
35	5	1
27	3	2
8 rd.	1 yd.	2 ft.

Write the subtrahend under the minuend, placing terms of the same denomination in the same column. Since 2 ft. are more than 1 ft., add 3 ft. to 1 ft., making 4 ft. for the term of the minuend, and take 2 ft. from 4 ft. and write 2 ft., the difference, under feet. Since 3 ft. were added to the minuend, add 1 yd. (which equals 3 ft.) to the 3 yd. of the subtrahend, making 4 yd., and take 4 yd. from 5 yd., and write 1 yd. (the difference) under yards. Subtract 27 rd. from 35 rd.. and write 8 rd. (the difference) under rods. The difference is 8 rd. 1 yd. 2 ft.

NOTE.—Instead of *adding* 1 yd. to 3 yd. before subtracting these terms, 1 yd. *may be taken* from the 5 yd. in the minuend. The teacher should show the similarity between the subtraction of compound numbers and simple numbers.

	2.			3.				4.			
	cwt.	lb.	oz.	gal.	qt.	pt.	gi.	w.	d.	h.	min.
From	48	73	10	44	3	1	2	13	1	13	45
Take	29	47	14	26	3	1	3	8	6	7	33

5. A farmer raised 7 bu. 1 pk. 4 qt. of clover-seed, and sold 5 bu. 6 qt. 1 pt.: how much had he left?

6. A note was given July 23, 1863, and paid Sept. 15, 1869: how long did it run?

PROCESS.

yr.	mo.	d.
1869	9	15
1863	7	23
6 yr.	1. mo.	22 d.

Write the earlier date under the later, writing the number of the year, month, and day in proper order, and subtract, allowing 30 days to a month, and 12 months to a year.

7. What is the difference of time between Oct. 23, 1856, and June 15, 1866?

8. What is the difference of time between April 12, 1861, to May 22, 1865?

9. Abraham Lincoln was born Feb. 12, 1809, and died April 15, 1865: what was his age?

10. James A. Garfield was born Nov. 19, 1831, and died September 19, 1881: what was his age?

11. The American Revolution began April 19, 1775, and ended Jan. 20, 1783: how long did it continue?

12. America was discovered Oct. 14, 1492, and the Declaration of Independence was signed July 4, 1776: how much time elapsed between these two events?

13. The laying of the Atlantic Cable was completed July 28, 1866, and the Pacific Railroad was completed May 10, 1869: how much earlier was the first event than the second?

14. Andrew Jackson died at Nashville, Tenn., June 8, 1845, aged 78 yr. 2 mo. 23 days: what was the date of his birth?

ART. 127. **Compound Subtraction** is the process of finding the difference between two compound numbers of the same kind.

LESSON XLII.

MULTIPLICATION OF COMPOUND NUMBERS.

WRITTEN EXERCISES.

1. Multiply 34 gal. 3 qt. 1 pt. by 9.

PROCESS.

gal.	qt.	pt.
34	3	1
		9
313 gal.	3 qt.	1 pt.

Reduce each successive product to the next higher denomination, and write the remainder, if any, under the term multiplied.

NOTE.—In both simple and compound multiplication *the successive products are divided by the number of units of each denomination which equals one of the next higher denomination.*

	2.			**3.**			**4.**				
	bu.	pk.	qt.	w.	d.	h.	mi.	rd.	yd.	ft.	in.
Multiply	27	3	5	4	6	13	150	140	0	2	11
By			9			7					6

5. John's age is 7 yr. 9 mo. 16 d., which is one fifth of the age of his father: how old is his father?

6. What is the distance round a square field, each side of which is 24 rd. 3 yd. 2 ft.?

7. If a man walk 2 mi. 80 rd. 3 yd. an hour, how far will he walk in 11 hours?

8. How many bushels of wheat will fill 24 sacks, each holding 3 bu. 2 pk. 5 qt.?

ART. 128. **Compound Multiplication** is the process of taking a compound number a given number of times.

LESSON XLIII.

DIVISION OF COMPOUND NUMBERS.

WRITTEN EXERCISES.

1. What is $\frac{1}{12}$ of 15 w. 6 d. 13 h. 12 min.?

PROCESS.

w.	d.	h.	min.
12)15	6	13	12
1 w.	2 d.	7 h.	6 min.

Reduce the successive remainders to lower denominations, adding to the result, in each case, the number of that denomination in the dividend.

NOTE.—When the divisor is a large number, the successive terms of the quotient may be written at the right of the dividend, as in long division.

2. What is $\frac{1}{6}$ of 19 cwt. 73 lb. 12 oz.?

3. If a man can dig a ditch 36 rd. 4 yd. 2 ft. long in 8 days, how much of it can he dig in 1 day?

4. A man bought a stack of hay containing 6 T. 19 cwt. 86 lb., and drew it home in 7 equal loads: how much hay did he draw at each load?

5. How many lengths of fence, each 10 ft. 4 in., will make 28 rd. 3 ft. of fence?

SUGGESTION.—Reduce both compound numbers to the same denomination (inches), and divide as in simple division. This involves the reduction of compound numbers to simple numbers.

6. How many castings, weighing 12 lb. 8 oz. each, can be made from 5 cwt. 50 lb. of iron?

7. How many baskets of peaches, containing 3 pk. 4 qt. each, will make $3\frac{1}{2}$ bushels?

8. How many bottles, holding 3 qt. 1 pt. each, can be filled from a cask containing $45\frac{1}{2}$ gallons?

ART. 129. **Compound Division** is the process of *dividing a compound* number into equal parts.

LESSON XLIV.

MISCELLANEOUS PROBLEMS.

ORAL PROBLEMS.

1. What will $5\frac{1}{2}$ quarts of plums cost at 6 cents a quart? At 8 cts. a quart? At 9 cts.?

2. A man carried $3\frac{3}{4}$ pecks of cherries to market, and sold them at 8 cents a quart: how much did he receive?

3. If a bushel of plums costing \$1.60 is sold at 6 cts. a quart, what is the gain?

4. When apples sell at 20 cents a peck, what are they worth a bushel?

5. A boy picked 3 pecks of cherries, and sold them at 10 cents a quart: how much did he receive?

6. What will 5 pk. 6 qt. of cherries cost at 5 cents a quart?

7. A boy bought half a bushel of chestnuts for \$1, and sold them at 8 cents a quart: how much did he make?

8. What will 10 quarts of milk cost at $5\frac{1}{2}$ cents a pint?

9. If a gallon of wine cost \$6, what will 1 pint cost?

10. If maple syrup cost \$1.60 a gallon, what is the price per quart?

11. At 6 cents a quart, what will 5 gallons of milk cost? $4\frac{1}{2}$ gallons? 10 gallons?

12. How many half-pint tumblers will 2 gallons of jelly fill?

13. How many quart baskets will 3 pk. 5 qt. of strawberries fill?

14. How many quarts in $\frac{5}{8}$ of a bushel? $\frac{3}{4}$ of a bushel?

15. How many pints in $3\frac{3}{4}$ gallons? $6\frac{1}{4}$ gallons?

16. How many days in $\frac{2}{7}$ of a week? $\frac{5}{7}$ of a week?

17. How many ounces in $2\frac{1}{2}$ pounds of sugar? $5\frac{1}{2}$ lb.? $6\frac{1}{4}$ lb.? $10\frac{3}{4}$ lb.?

18. What will $\frac{2}{5}$ of a cwt. of sugar cost at 10 cents a pound? At $12\frac{1}{2}$ cts.? At 15 cts.? At 11 cts.?

19. What will $\frac{3}{8}$ of a gallon of oil cost at 25 cents a pint? At $33\frac{1}{3}$ cts. a pint? At $37\frac{1}{2}$ cts.?

20. What will $\frac{2}{5}$ of a ream of paper cost at 20 cents a quire? At $12\frac{1}{2}$ cts.? At 15 cts.?

21. What will $\frac{2}{5}$ of a ton of hay cost at 75 cents a cwt.? At 90 cts.? At $1?

22. How many square yards in 36 square feet?

23. How many square feet in $5\frac{1}{3}$ sq. yd.?

24. What part of a pound of sugar is 8 oz.? 9 oz.? 12 oz.? 4 oz.? 11 oz.? 13 oz.? 20 oz.?

25. What part of a foot is 3 inches? 4 in.? 6 in.? 8 in.? 9 in.? 10 in.? 15 in.? 18 in.?

26. What part of a yard is 6 inches? What part is 12 inches? 18 inches?

27. How many inches in $\frac{3}{4}$ of a yard? In $\frac{2}{3}$ yd.?

28. How many steps, each 2 ft. 6 in., will a man take in crossing a street 50 ft. wide?

SUGGESTION.—Reduce both numbers to inches or to half-feet.

29. How many times will a hoop 5 feet in circumference turn round in rolling 25 yards?

30. How many times will a wheel 10 feet in circumference turn round in running 50 yards?

31. A sleigh is 7 feet long; how many times its length will it run in 35 yards? In 70 yards?

32. What will 20 yards of lead pipe cost at $12\frac{1}{2}$ cts. a foot? At 15 cts. a foot?

33. How many barrels, each holding 2 bu. 3 pk., will it take to pack 33 bushels of apples?

34. A sack of wheat weighs 210 pounds: how many *bushels in it?*

35. If a ship sail 3 leagues an hour, in how many hours will it sail 63 miles? 90 miles?

36. How many leap years in this century?

37. How many calendar months in 10 years?

38. What will $2\frac{1}{2}$ bushels of strawberries cost at 10 cts. a quart? At $12\frac{1}{2}$ cts.?

39. What will 12 ounces of butter cost at 20 cts. a pound? At 25 cts.? At 30 cts.? At 35 cts.?

40. What will 3 lb. 8 oz. of beefsteak cost at 12 cts. a pound? At 10 cts.? At 15 cts.?

41. What will 75 lbs. of hay cost at 80 cts. a hundred?

42. What will 6 ounces of nutmegs cost at 40 cts. a pound?

WRITTEN PROBLEMS.

43. How many quarts of cranberries in 5 barrels, each containing 2 bu. 3 pk.?

44. How many pints of vinegar in 12 barrels, each containing $31\frac{1}{2}$ gallons?

45. How many yards in $16\frac{3}{8}$ miles?

46. How many square yards in 6 acres of land?

47. How many cubic yards in 1215 cu. ft. of stone?

48. How many minutes in the month of February, 1882? In February, 1884?

49. What is the cost of a stack of hay containing 9000 pounds, at $12 a ton?

50. A fruit dealer bought 12 bushels of strawberries at $2.25 a bushel, and sold them at 10 cts. a quart: how much did he gain?

51. What will 20 yd. 2 ft. of iron railing cost at $1.25 a foot? At $1.37\frac{1}{2}$ a foot?

52. What will 40 miles of telegraph wire cost at 75 cents a rod? At $87\frac{1}{2}$ cts. a rod?

53. What will it cost to transport 50 T. 15 cwt. 75 lb. *of freight, at* $\frac{1}{2}$ cent a pound?

54. A farmer exchanged $45\frac{3}{4}$ pounds of butter, at 20 cents a pound, for sugar at 15 cents a pound: how much sugar did he receive?

55. A steamer sails 3 leagues an hour: how many hours will it take it to sail 180 miles? 260 miles?

56. How many times will a carriage-wheel 11 feet in circumference turn round in running 2.5 miles?

57. How many times will a car-wheel 5 feet in circumference turn round in running from Columbus to Cincinnati, the distance being 120 miles?

58. How many steps of 2 ft. 6 in. each will a man take in walking 2 miles? In walking $\frac{3}{4}$ of a mile?

59. How many times will a wheel 6 feet in circumference turn round in going $2\frac{1}{2}$ miles? 4 miles?

60. Sound travels 1090 feet a second: how many miles will it travel in 60 seconds?

61. How many rods of fence will be required to inclose a farm which is $\frac{1}{4}$ of a mile long and $\frac{1}{8}$ of a mile wide?

62. How many times will a clock that ticks seconds tick in the month of June? In the month of July?

63. If a person read a half hour each day, how many hours will he read in 40 years, of $365\frac{1}{4}$ days each?

64. A car contains 80 barrels of pork, and another car 80 barrels of flour: what is the difference in the number of pounds of freight in the two cars?

65. How many gross of pens will supply 4320 pupils one year, if each pupil require 4 pens?

66. If 10 sheets of paper will make a 16mo. book of 320 pages, how many reams will it take to publish an edition of 2000 copies?

67. A fruit dealer bought 24 barrels of apples, containing $2\frac{3}{4}$ bushels each, at \$2.50 a barrel, and sold them at \$1.25 a bushel: what was his gain?

68. A grocer bought 6 barrels of vinegar, containing $31\frac{1}{2}$ gallons each, at \$6.50 a barrel, and sold it at 10 *cents a quart*: how much did he make?

69. A merchant bought a hogshead of molasses, containing 63 gallons, and sold $\frac{2}{3}$ of it at 75 cents a gallon, and the rest at 20 cents a quart: what did he receive for it?

70. A man bought 5 hogsheads of molasses, each containing 63 gallons, at \$31.50 a hogshead, and sold 3 hogsheads at 70 cents a gallon, and 2 hogsheads at 65 cents a gallon: how much did he gain?

71. A grocer bought a car-load of potatoes, weighing 15000 lb., at 65 cts. a bushel, and sold them at 80 cts. a bushel: what was the cost of the load? How much did he gain?

72. What will be the cost of a load of hay weighing 2750 pounds at \$12.50 a ton? At \$10.25 a ton?

73. How many steps, each 2 ft. 4 in., will a man take in walking $1\frac{1}{2}$ miles? $2\frac{1}{4}$ miles?

74. How many jugs, each holding 1 gal. 2 qt., can be filled from a barrel of vinegar containing $31\frac{1}{2}$ gallons?

75. A grocer bought 25 gallons of maple syrup at \$1.20 a gallon, and sold it at 40 cts. a quart: how much did he gain?

76. If 5 sheets of copper contain 28 lb. 10 oz., how much copper is there in each sheet?

77. Six equal casks of vinegar contain 218 gal. 2 qt.: how much vinegar in each cask?

78. If a milk dealer sell daily 7 cans of milk, each holding 12 gal. 2 qt., how much milk does he sell in 4 weeks?

79. From the sum of 15 lb. 8 oz. and 9 lb. 10 oz. take their difference.

80. John Jones was born Aug. 8, 1866, and on Jan. 1, 1882, his age was just $\frac{1}{4}$ of the age of his father: how old was his father?

81. President Lincoln was assassinated April 14, 1865, and President Garfield was assassinated July 2, 1881: *how much time* between the two events?

82. A farmer having cut 12 T. 15 cwt. of hay from a meadow, sold 6 loads of 1 T. 3 cwt. 75 lb. each, and then put the rest in a stack: how much hay did he sell? How much hay was in the stack?

83. A merchant bought 3 chests of tea, each weighing 2 cwt. 45 lb., and in one month sold 4 cwt. 80 lb. 12 oz.: how much tea had he left?

84. A publisher bought 20 bundles of paper, and used daily 3 reams 15 quires 12 sheets: how much paper had he left at the close of 10 days?

85. A railroad company bought 145 cords of wood, piled in 3 ranks; the first rank contained 36 cd. 80 cu. ft., and the second, 64 cd. 96 cu. ft.: how much wood was in the third rank?

86. A man bought 3 loads of hay, which, with the wagon, weighed respectively 1 T. 8 cwt. 40 lb.; 1 T. 11 cwt. 80 lb.; and 1 T. 9 cwt. 60 lb.; and the wagon alone weighed 10 cwt. 90 lb.: how much hay did he buy? What was its cost at $8.50 a ton?

QUESTIONS FOR REVIEW.

What is a number? What is an abstract number? A concrete number? When is a concrete number called *denominate?* Give a concrete number that is denominate. Give one that is not denominate.

Into what two classes are denominate numbers divided? Define a simple denominate number. Define a compound denominate number. Why may compound denominate numbers be called briefly compound numbers? Is every compound number denominate? Why? Give a denominate number that is simple.

What is United States Money? What is currency? Of how many kinds of money is the currency of the United States composed? What are the principal gold coins? What are the silver coins now coined by the United States? What two coins are made of copper and nickel? Of what is the two-cent piece and the cent composed? Is there such a coin as a mill? What part *of a cent is a mill?* Name the two kinds of paper money in

circulation. What coin is called a "dime"? What coin is called a "nickel"? What coin is called a "quarter"? What coin is called an "eagle"? What coin is called a "double-eagle"?

What is meant by measure? Name the two kinds of measures. How are the measures of extension divided?

What table is used in measuring lines? Surfaces? Solid contents? Capacity? What table is used in measuring duration? What ones are used in measuring the weight of bodies?

What is reduction? Name the two kinds of reduction. Define each kind. Repeat the rule for each.

For what is Dry Measure used? Name the denominations. Repeat the table. For what is Liquid Measure used? Name the denominations. Repeat the table. For what is Long Measure used? Name the denominations. Repeat the table.

For what is Square Measure used? Name the denominations. Repeat the table. What is a square inch? A square yard? For what is Cubic Measure used? Name the denominations. Repeat the table. What is a cubic inch? A cubic yard?

For what is time measure used? Name the denominations. Repeat the table.

Name the calendar months in their order, and give the number of days in each. How many days has February in leap years? Name the four seasons of the year, and the months of each.

For what are the three kinds of weights respectively used? Which of these is in common use? Give the denominations, and repeat the table. Repeat the miscellaneous table.

What is a compound number? What are the successive denominations of a compound number called? When are compound numbers of the same kind? When can two or more compound numbers be added? In what respect does compound addition differ from simple addition?

Define compound subtraction. In what respect does it differ from simple subtraction?

Define compound multiplication. In what respect does it differ from simple multiplication? Define compound division.

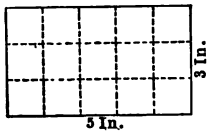
MENSURATION.

LESSON XLV.

MEASUREMENT OF SURFACES.

ORAL PROBLEMS.

1. How many square inches in a piece of paper 5 inches long and 3 inches wide?



SOLUTION.—In a piece 5 inches long and 1 inch wide there are 5 sq. in.; and in a piece 3 inches wide there are 3 times 5 sq. in., which is 15 sq. in.

2. How many square inches in a piece of tin 10 inches long and 7 inches wide? 9 inches wide?

3. How many square feet in a piece of zinc 9 feet long and 5 feet wide? 11 ft. long and 6 ft. wide?

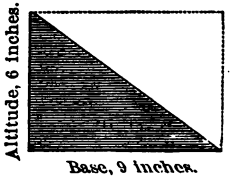
4. How many square feet in a pavement 20 yards long and 3 yards wide? 30 yd. long and 5 yd. wide?

5. How many square yards in a floor 8 yards long and 6 yards wide? 10 yd. long and $7\frac{1}{2}$ yd. wide?

6. How many square feet in a board 16 feet long and $1\frac{1}{4}$ feet wide? 12 ft. long and $\frac{3}{4}$ ft. wide?

7. How many square yards in a ceiling 10 yards long and 8 yards wide? 12 yd. long and 7 yd. wide?

8. How many square inches in a triangular piece of tin whose base is 9 inches and altitude (height) 6 inches.



SOLUTION.—Since the area of a triangle is one half the area of a rectangle with the same base and altitude, the piece of tin contains one half of 6 times 9 sq. in., or $\frac{1}{2}$ of 54 sq. in., which *is 27 sq. in.*

9. How many square feet in a triangular flower bed whose base is 20 feet and altitude 10 feet?

10. How many square rods in a triangular garden whose base is 12 rods and altitude 10 rods?

WRITTEN PROBLEMS.

11. How many square yards in a garden 66 yards long and $60\frac{1}{2}$ yards wide?

12. How many square feet in a ceiling 45 feet long and $33\frac{1}{3}$ feet wide? How many square yards?

13. How many square rods in a field 56 rods long and 40 rods wide? How many acres?

PROCESS. (11)

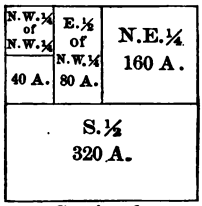
66 sq. yd.
$60\frac{1}{2}$
3960
33
3993 sq. yd.

14. How many acres in a field 60 rods long and 32 rods wide? 48 rods wide? 60 rods wide?

15. How many acres in a street $2\frac{1}{2}$ miles long and 4 rods wide? 10 rods wide?

16. How many acres in a half-section of land? In a quarter-section?

NOTE.—Townships laid out by the United States Government, since 1800, contain 36 square miles (6 miles square), and each square mile is called a **Section**. Each section (640 acres) is divided into *half-sections, quarter-sections, half-quarter-sections*, etc. The diagram shows how a section is divided and numbered.



Section 1.

17. How many acres of land in a quarter-section and a half-quarter-section together?

ART. 130. 1. To find the area of a rectangle:

Rule.—*Multiply the length by the width.*

2. To find the area of a triangle:

Rule.—*Multiply the base by one half of the altitude.*

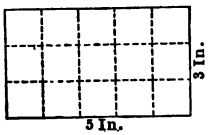
MENSURATION.

LESSON XLV.

MEASUREMENT OF SURFACES.

ORAL PROBLEMS.

1. How many square inches in a piece of paper 5 inches long and 3 inches wide?



SOLUTION.—In a piece 5 inches long and 1 inch wide there are 5 sq. in.; and in a piece 3 inches wide there are 3 times 5 sq. in., which is 15 sq. in.

2. How many square inches in a piece of tin 10 inches long and 7 inches wide? 9 inches wide?

3. How many square feet in a piece of zinc 9 feet long and 5 feet wide? 11 ft. long and 6 ft. wide?

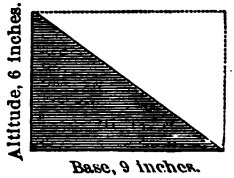
4. How many square feet in a pavement 20 yards long and 3 yards wide? 30 yd. long and 5 yd. wide?

5. How many square yards in a floor 8 yards long and 6 yards wide? 10 yd. long and $7\frac{1}{2}$ yd. wide?

6. How many square feet in a board 16 feet long and $1\frac{1}{4}$ feet wide? 12 ft. long and $\frac{3}{4}$ ft. wide?

7. How many square yards in a ceiling 10 yards long and 8 yards wide? 12 yd. long and 7 yd. wide?

8. How many square inches in a triangular piece of tin whose base is 9 inches and altitude (height) 6 inches.



SOLUTION.—Since the area of a triangle is one half the area of a rectangle with the same base and altitude, the piece of tin contains one half of 6 times 9 sq. in., or $\frac{1}{2}$ of 54 sq. in., which *is 27 sq. in.*

II. Plastering and Painting.

How many square yards of plastering in the four
of a room 18 ft. long, 15½ ft. wide, and 10 ft.
ith no allowance for doors and windows?

Plastering, kalsomining, and painting are measured by
yard. There is no uniform rule respecting the allow-
ade for doors and windows.
of the walls of a room may be found *by adding four sides and multiplying the sum by the height.*

6. How many square yards of plastering in the ceiling of a room 16⅔ ft. long, 15 ft. wide, and 9 ft. high? How many square yards in walls and ceiling?

8. How many square yards of plastering in the walls and ceiling of a room 16 ft. long, 14⅓ ft. wide, and 9 ft. high, with an allowance of 12 sq. yd. for doors and windows? What will be the cost of plastering at 25 cts. a square yard?

9. What will it cost to paint the walls and ceiling of a parlor 20½ ft. long, 16 ft. wide, and 12 ft. high, at 12½ cts. a square yard, allowing 15 square yards for doors and windows?

10. What will be the cost of kalsomining a hall 50 ft. long, 42½ ft. wide, and 14 ft. high, at 5 cts. a square yard, allowing 16 square yards for doors and windows?

Note.—Glazing and stone-cutting are measured by the square foot; flooring, roofing, tiling, and paving, by the square of 100 feet. Paving is also estimated by the square yard and by the 1000 bricks.

III. Board Measure.

11. How many feet of lumber in a board 16 ft. long, 9 in. wide, and 1 in. thick? ½ in. thick? 1¼ in. thick?

Process: 16 sq. ft. $\times \frac{3}{4} \times 1\frac{1}{4} = 15$ sq. ft., Ans. to last.

LESSON XLVI.

APPLICATIONS OF SURFACE MEASUREMENTS.

I. CARPETING ROOMS.

1. How many yards of carpeting 1 yd. wide will carpet a floor 27 feet long and 21 feet wide?

NOTE.—The number of yards of carpeting required to carpet a room depends on the direction in which the strips run, whether lengthwise or across the room, and how much must be turned under or cut off at the side or end of the room or both. The proper matching of the figures may require the turning under or cutting off the carpet at one end. The number of yards in each strip multiplied by the number of strips required will give the number of yards. Carpeting is sold by **Long Measure.**

2. How many yards of carpeting $\frac{3}{4}$ of a yard wide will carpet a floor 18 ft. long and 12 ft. wide, if the strips run lengthwise and there be no loss from matching figures? How many yards will be required if the strips run across the room and there be no loss from matching figures?

3. How many yards of carpeting 1 yd. wide will cover a floor $16\frac{1}{3}$ ft. long and $14\frac{1}{2}$ ft. wide, if the strips run lengthwise and the matching of the figures require 8 inches to be turned under at one end of the room?

SUGGESTION.—It will require 5 strips of carpeting each 17 feet long.

4. How many yards of Brussels carpeting $\frac{3}{4}$ yd. wide will carpet a parlor 20 ft. long and $15\frac{3}{4}$ ft. wide, if the strips run lengthwise and the matching of the figures require 6 inches to be turned under?

5. How many yards of matting $1\frac{1}{4}$ yd. wide will cover the floor of a hall 60 ft. long and 15 ft. wide, if the strips run lengthwise? How many yards of matting $\frac{3}{4}$ yd. wide?

II. Plastering and Painting.

6. How many square yards of plastering in the four walls of a room 18 ft. long, 15½ ft. wide, and 10 ft. high, with no allowance for doors and windows?

Note.—Plastering, kalsomining, and painting are measured by the square yard. There is no uniform rule respecting the allowance to be made for doors and windows.

The surface of the walls of a room may be found *by adding the lengths of the four sides and multiplying the sum by the height.*

7. How many square yards of plastering in the ceiling of a room 16⅔ ft. long, 15 ft. wide, and 9 ft. high? How many square yards in walls and ceiling?

8. How many square yards of plastering in the walls and ceiling of a room 16 ft. long, 14⅓ ft. wide, and 9 ft. high, with an allowance of 12 sq. yd. for doors and windows? What will be the cost of plastering at 25 cts. a square yard?

9. What will it cost to paint the walls and ceiling of a parlor 20½ ft. long, 16 ft. wide, and 12 ft. high, at 12½ cts. a square yard, allowing 15 square yards for doors and windows?

10. What will be the cost of kalsomining a hall 50 ft. long, 42½ ft. wide, and 14 ft. high, at 5 cts. a square yard, allowing 16 square yards for doors and windows?

Note.—Glazing and stone-cutting are measured by the square foot; flooring, roofing, tiling, and paving, by the square of 100 feet. Paving is also estimated by the square yard and by the 1000 bricks.

III. Board Measure.

11. How many feet of lumber in a board 16 ft. long, 9 in. wide, and 1 in. thick? ½ in. thick? 1¼ in. thick?

Process: 16 sq. ft. $\times \frac{3}{4} \times 1\frac{1}{4} = 15$ sq. ft., Ans. to last.

NOTE.—Boards 1 inch or less in thickness are sold by the square foot, surface measure. Boards more than 1 inch in thickness, joists, sills, and all other square lumber, are measured *by multiplying the number of square feet in one surface by the thickness in inches.*

12. How many feet of lumber in 16 joists, each 16 ft. by 12 in. by 2 in.?

13. How many feet of lumber in 20 scantlings, each 16 ft. by 4 in. by 2 in.?

14. How many feet of lumber in an inch board 14 ft. long and 8 in. wide at one end and 4 in. wide at the other?

NOTE.—The average width of the board is $\frac{1}{2}$ of the sum of 8 inches and 4 inches, which is 6 inches, or $\frac{1}{2}$ foot. In practice, this would be found by measuring the width of the board at its middle.

15. How many feet of lumber in 5 boards, each 12 ft. long, 10 in. wide at one end and 2 in. wide at the other, and $1\frac{1}{2}$ in. thick?

16. How many feet of lumber in 15 boards, each 12 ft. long and 7 in. wide, and 21 boards, each 14 ft. long and 8 in. wide, each board being 1 in. thick?

17. How many feet of timber in 12 joists, 2 in. by 8 in., and 15 ft. long, and 20 scantling, 3 in. by 4 in., and 12 ft. long?

ART. 131. 1. To find the number of feet of lumber in a board 1 inch or less in thickness:

Rule.—*Multiply the length in feet by the width in inches, and divide the product by 12.*

2. To find the number of feet in lumber more than 1 inch in thickness:

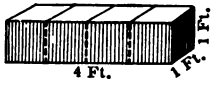
Rule.—*Multiply the number of square feet in one surface by the thickness in inches.*

LESSON XLVII.

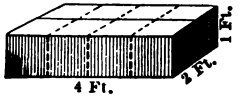
MEASUREMENTS OF SOLIDS.

ORAL PROBLEMS.

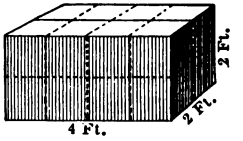
1. How many cubic feet in a block of wood 4 ft. long, 1 ft. wide, and 1 ft. thick?



2. How many cubic feet in a block 4 ft. long, 2 ft. wide, and 1 ft. thick?



3. How many cubic feet in a block 4 ft. long, 2 ft. wide, and 2 ft. thick?



SOLUTION.—A block 4 ft. long, 1 ft. wide, and 1 ft. thick contains 4 cu. ft.; a block 4 ft. long, 2 ft. wide, and 1 ft. thick contains 2 times 4 cu. ft., which is 8 cu. ft.; and a block 4 ft. long, 2 ft. wide, and 2 ft. thick contains 2 times 8 cu. ft., which is 16 cu. ft.

4. How many cubic feet in a block 4 ft. long, 3 ft. wide, and 2 ft. thick?

5. How many cubic feet in a cubic block 3 ft. long, 3 ft. wide, and 3 ft. thick?

6. How many cubic inches in a brick 8 in. long, 4 in. wide, and 2 in. thick?

7. How many cubic inches in a block of silver 6 in. long, 2 in. wide, and $1\frac{1}{2}$ in. thick?

8. How many cubic feet in a box 5 ft. long, 4 ft. wide, and 3 ft. deep?

9. How many cubic feet in a pile of wood 10 ft. long, 4 ft. wide, and 5 ft. high?

10. How many cubic feet in a brick wall 50 ft. long, 4 ft. high, and 2 ft. thick?

WRITTEN PROBLEMS.

11. How many cubic feet in a block of marble 15 ft. long, 12 ft. wide, and 7 ft. thick?

A block 15 ft. by 1 ft. by 1 ft. contains 15 cu. ft.; a block 15 ft. by 12 ft. by 1 ft. contains 12 times 15 cu. ft., or 180 cu. ft.; a block 15 ft. by 12 ft. by 7 ft. contains 7 times 180 cu. ft., which is 1260 cu. ft.

PROCESS.

15 cu. ft.
12
180 cu. ft.
7
1260 cu. ft.

12. How many cubic feet in a rectangular rock 18 ft. long, 13 ft. wide, and 8 ft. high?

13. How many cubic feet in a pile of wood 24 ft. long, 6 ft. wide, and 8 ft. high?

14. How many cubic feet in a cellar 44 ft. long, 27 ft. wide, and 5 ft. deep? How many cubic yards?

15. How many cubic feet of water does a cistern hold which is 5 feet square and 3 ft. 4 in. deep?

16. How many cubic feet in a block of marble 10 ft. long, 20 in. wide, and 14 in. thick?

17. How many cubic feet of air in a school-room 36 ft. long, 25 ft. wide, and 12 ft. high? If there are 53 pupils and 1 teacher in the room, how many cubic feet of air to each person?

18. How many cubic yards of earth must be removed to excavate a cellar 36 ft. long, $28\frac{1}{2}$ ft. wide, and 5 ft. deep? What will be the cost at $33\frac{1}{3}$ cts. a cubic yard?

ART. 132. To find the solid contents of a rectangular solid:

Rule.—*Multiply the length, width, and thickness together.*

NOTE.—The length is first changed to cubic units, and the *width and thickness* are used as abstract numbers.

LESSON XLVIII.

APPLICATIONS OF SOLID MEASUREMENTS.

I. WOOD MEASURE.

1. How many cords of wood in a pile containing 1536 cu. ft.? 1920 cu. ft.? 1728 cu. ft.?

2. How many cords of wood in a pile 20 ft. long, 4 ft. wide, and 6 ft. high? 5 ft. high?

NOTE.—A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high contains 1 cord, and 1 foot in length of such a pile contains 16 cu. ft., or 1 *cord foot* (little used).

3. How many cords of wood in a pile 48 ft. long, 4 ft. wide, and $5\frac{1}{2}$ ft. high? 6 ft. high?

4. How many cords of four-foot wood in a pile 64 ft. long and $5\frac{1}{2}$ ft. high? $7\frac{1}{2}$ ft. high?

NOTE.—Wood four feet in length, or less, is usually measured *by multiplying the length of the pile by the height, and dividing the product by 32.*

5. How many cords of three-foot wood in a pile 28 ft. long and 6 ft. high? 24 ft. long and $7\frac{1}{2}$ ft. high?

82. A farmer having cut 12 T. 15 cwt. of hay from a meadow, sold 6 loads of 1 T. 3 cwt. 75 lb. each, and then put the rest in a stack: how much hay did he sell? How much hay was in the stack?

83. A merchant bought 3 chests of tea, each weighing 2 cwt. 45 lb., and in one month sold 4 cwt. 80 lb. 12 oz.: how much tea had he left?

84. A publisher bought 20 bundles of paper, and used daily 3 reams 15 quires 12 sheets: how much paper had he left at the close of 10 days?

85. A railroad company bought 145 cords of wood, piled in 3 ranks; the first rank contained 36 cd. 80 cu. ft., and the second, 64 cd. 96 cu. ft.: how much wood was in the third rank?

86. A man bought 3 loads of hay, which, with the wagon, weighed respectively 1 T. 8 cwt. 40 lb.; 1 T. 11 cwt. 80 lb.; and 1 T. 9 cwt. 60 lb.; and the wagon alone weighed 10 cwt. 90 lb.: how much hay did he buy? What was its cost at $8.50 a ton?

QUESTIONS FOR REVIEW.

What is a number? What is an abstract number? A concrete number? When is a concrete number called *denominate?* Give a concrete number that is denominate. Give one that is not denominate.

Into what two classes are denominate numbers divided? Define a simple denominate number. Define a compound denominate number. Why may compound denominate numbers be called briefly compound numbers? Is every compound number denominate? Why? Give a denominate number that is simple.

What is United States Money? What is currency? Of how many kinds of money is the currency of the United States composed? What are the principal gold coins? What are the silver coins now coined by the United States? What two coins are made of copper and nickel? Of what is the two-cent piece and the cent composed? Is there such a coin as a mill? What part *of a cent is a mill?* Name the two kinds of paper money in

circulation. What coin is called a "dime"? What coin is called a "nickel"? What coin is called a "quarter"? What coin is called an "eagle"? What coin is called a "double-eagle"?

What is meant by measure? Name the two kinds of measures. How are the measures of extension divided?

What table is used in measuring lines? Surfaces? Solid contents? Capacity? What table is used in measuring duration? What ones are used in measuring the weight of bodies?

What is reduction? Name the two kinds of reduction. Define each kind. Repeat the rule for each.

For what is Dry Measure used? Name the denominations. Repeat the table. For what is Liquid Measure used? Name the denominations. Repeat the table. For what is Long Measure used? Name the denominations. Repeat the table.

For what is Square Measure used? Name the denominations. Repeat the table. What is a square inch? A square yard? For what is Cubic Measure used? Name the denominations. Repeat the table. What is a cubic inch? A cubic yard?

For what is time measure used? Name the denominations. Repeat the table.

Name the calendar months in their order, and give the number of days in each. How many days has February in leap years? Name the four seasons of the year, and the months of each.

For what are the three kinds of weights respectively used? Which of these is in common use? Give the denominations, and repeat the table. Repeat the miscellaneous table.

What is a compound number? What are the successive denominations of a compound number called? When are compound numbers of the same kind? When can two or more compound numbers be added? In what respect does compound addition differ from simple addition?

Define compound subtraction. In what respect does it differ from simple subtraction?

Define compound multiplication. In what respect does it differ from simple multiplication? Define compound division.

14. What will it cost to pave a walk 60 feet long and 15 feet wide, at $1.25 a square yard?

15. What will it cost to gravel a street 129 rods long and 60 feet wide, at 75 cents a square yard?

16. How many square yards in the walls and ceiling of a room 21 ft. long, 18 ft. wide, and 9 ft. high?

17. How many square yards in a ceiling 40½ ft. long and 36 ft. wide? 29 ft. long and 22½ ft. wide?

18. How many yards of carpeting a yard wide will carpet a room 18⅓ ft. long and 15 ft. wide, if the strips run lengthwise, with no allowance for matching figures?

19. If 1000 shingles will cover 100 square feet, how many shingles will cover a roof, each of the two sides of which is 48 ft. long and 15 ft. wide?

20. A garden contains 185 square rods, and is 15 rods long: how wide is it?

21. How many cubic ft. in a bin 12 ft. long, 8 ft. wide, and 3½ ft. deep?

22. At $4.50 a cord, what will be the cost of a pile of four-foot wood 48 ft. long and 6 ft. high?

23. What will it cost to dig a cellar 36 ft. long, 18¾ ft. wide, and 6½ ft. deep, at $2.50 a cubic yard?

24. How many bushels of wheat will a bin hold that is 6 ft. long, 4½ ft. wide, and 2½ ft. deep?

25. How many gallons of water will a tank hold that is 5 ft. long, 4 ft. wide, and 2¾ ft. deep?

QUESTIONS FOR REVIEW.

What is the rule for finding the area of a rectangle? Of a triangle?

Give the rule for measuring lumber 1 inch or less in thickness; also when more than 1 inch in thickness.

Give the rule for finding the contents of a rectangular solid.

How is the capacity of a bin in bushels found?

How is the capacity of a tank in gallons found? In barrels?

PERCENTAGE.

LESSON L.

ORAL EXERCISES.

ART. 133. One hundredth of a number is one per cent of it; two hundredths, two per cent; three hundredths, three per cent, and so on.

1. What per cent of a number is 5 hundredths of it? 7 hundredths of it? 10 hundredths?

2. What per cent of a number is $\frac{4}{100}$ of it? $\frac{9}{100}$ of it? $\frac{15}{100}$ of it? $\frac{75}{100}$ of it? $\frac{100}{100}$ of it?

3. What per cent of a number is .05 of it? .13 of it? .35 of it? .06 of it? .06$\frac{1}{2}$ of it?

4. What per cent of a number is .12$\frac{1}{2}$ of it? .33$\frac{1}{3}$ of it? .03$\frac{1}{3}$ of it? .00$\frac{1}{3}$ of it? .00$\frac{1}{2}$ of it?

5. How many hundredths of a number is 6 per cent of it? 15 per cent of it? 5$\frac{1}{2}$ per cent of it? 6$\frac{3}{8}$ per cent of it?

NOTE.—The character %, called the per cent sign, is used for the words *per cent.* Thus, 5% denotes 5 per cent, and 3$\frac{1}{3}$% denotes 3$\frac{1}{3}$ per cent.

6. How many hundredths of a number is 7% of it? 6$\frac{1}{2}$% of it? 12$\frac{1}{2}$%? $\frac{1}{2}$%? 6$\frac{3}{4}$%? $\frac{3}{4}$%?

7. How many hundredths of a number is 75% of it? 90%? 100%? 125%? 150%?

ART. 134. Any per cent of a number may be expressed as a *common fraction* or as a *decimal.* Thus:

6	per	cent,	or	6%,	may	be	expressed	as	$\frac{6}{100}$,	or	as	.06
4	"	"	"	4%,	"	"	"	"	$\frac{4}{100}$,	"	"	.04
5	"	"	"	5%,	"	"	"	"	$\frac{5}{100}$,	"	"	.05
12	"	"	"	12%,	"	"	"	"	$\frac{12}{100}$,	"	"	.12

WRITTEN EXERCISES.

8. Express decimally 4%; 3%; 12%; 15%; 20%.
9. Express decimally 11%; 30%; 25%; 40%; 18%.
10. Express decimally 70%; 85%; 125%; 150%.

PROCESS: $85\% = .85$; $125\% = 1.25$; $150\% = 1.50$.

11. Express decimally 45%; 77%; 80%; 115%; 175%.
12. Express decimally 5%; 7%; 16%; 24%; 40%.
13. Express decimally $\frac{1}{2}\%$; $\frac{3}{8}\%$; $\frac{3}{7}\%$; $\frac{5}{12}\%$; $\frac{1}{80}\%$.
14. Express decimally $12\frac{1}{2}\%$; $16\frac{2}{3}\%$; $62\frac{1}{4}\%$; $17\frac{1}{10}\%$.

SUGGESTION.—$12\frac{1}{2}\% = .12\frac{1}{2}$, or .125; $62\frac{1}{4}\% = .62\frac{1}{4}$, or .6225

15. Express decimally $\frac{1}{3}\%$; $\frac{3}{5}\%$; $\frac{1}{4}\%$; $\frac{3}{7}\%$; $\frac{7}{15}\%$.
16. Express decimally $7\frac{3}{10}\%$; $20\frac{1}{4}\%$; $112\frac{1}{2}\%$; $133\frac{1}{3}\%$.

DEFINITIONS.

ART. 135. A **Percentage** of a number is the result obtained by taking a given per cent of it.

The term per cent is a contraction of the Latin *per centum*, which means *by the hundred*. Any per cent of a number is so many hundredths of it.

The **Rate Per Cent** is the number of hundredths taken.

NOTE.—The term *rate per cent* denotes the fraction which is the number of hundredths, and the term *rate* denotes the numerator of the fraction. Thus, $\frac{5}{100}$ is a rate per cent, and 5 the rate.

ART. 136. The character % is the **Per Cent Sign**, and is read *per cent*.

ART. 137. **Percentage** embraces all numerical operations *in which one hundred* is the basis.

LESSON LI.

TO FIND A GIVEN PER CENT OF ANY NUMBER.

ORAL EXERCISES.

1. How much is 5 per cent of 200?

SOLUTION.—5% of 200 is $\frac{5}{100}$ of 200: $\frac{1}{100}$ of 200 is 2, and $\frac{5}{100}$ of 200 is 5 times 2, which is 10: 5% of 200 is 10.

2. What is 4 per cent of 300? Of 400? 500? 800?
3. What is 5 per cent of 200? Of 500? 600? 900?
4. What is 6 per cent of 150? Of 250? 450? 700?
5. What is 10 per cent of \$300? \$500? \$600? \$50?
6. What is 8 per cent of \$50? \$25? \$40? \$100?
7. What is 1% of 300? 2%? 4%? 6%? 8%?

SOLUTION.—1% of 300 is 3, and 2% is twice 3, which is 6, etc.

8. What is 1% of \$500? 3%? 5%? 7%? 9%?
9. What is 1% of \$1200? $\frac{1}{2}$%? $\frac{1}{4}$%? $\frac{3}{4}$%? $\frac{1}{8}$%?
10. What is 5% of 300 bu.? 10%? 3%? $\frac{1}{2}$%?
11. What is 6% of 1000? Of 2000? 5000? 2500?

TO TEACHERS.—Show that, instead of finding $\frac{1}{100}$ or 1% of 2500, and multiplying the result by 6, the pupil may multiply 2500 by .06, using slate or board. Let the pupil solve the following examples both orally (by finding 1%, etc.) and on the slate, by multiplying by the rate per cent expressed decimally.

12. What is 6% of \$200? \$250? \$400? \$450?
13. What is 5% of 340 yr.? 460 yr.? 650 yr.? 84 yr.?
14. What is 4% of 55 lb.? 450 lb.? 550 lb.? 600 lb.?
15. What is 10% of \$300? \$360? \$760? \$870?
16. What is 4 per cent of \$150? 6% of \$200? 8% of \$500? 12% of \$60? 10% of \$800? 7% of \$300?
17. What is 3 per cent of 50 bu.? 4% of 200 bu.? 8% of 500 lb.? 10% of \$600? $\frac{1}{2}$% of 64 ft.? $\frac{1}{8}$% of 90 *yd.*?

WRITTEN EXERCISES.

18. What is 16% of 324? $5\frac{1}{8}\%$ of \$724.50?

19. What is 8% of \$3250?

20. 6 % of 245?

21. 9 % of 1200?

22. 15 % of 644?

23. 5 % of 1540?

24. 10 % of 1050 ft.?

25. 33 % of 560 lb.?

26. $3\frac{1}{3}\%$ of 321 oz.?

PROCESS. (18.)

\$724.50

$.05\frac{1}{8}$

24150

362250

\$38.6400, *Ans.*

27. $12\frac{1}{2}\%$ of 960 men?

28. $\frac{1}{2}\%$ of \$450?

29. $\frac{3}{5}\%$ of \$525?

30. $\frac{2}{3}\%$ of 365 days?

31. $\frac{3}{10}\%$ of \$9650?

32. 6% of $\$.62\frac{1}{2}$?

33. 8% of 6.45?

34. 3% of 40.5 ft.?

35. $6\frac{1}{8}\%$ of 96.6 miles?

36. $2\frac{1}{5}\%$ of 120 acres?

37. 10 % of 78.4 lb.?

38. $\frac{3}{10}\%$ of 160 days?

39. $1\frac{2}{5}\%$ of 250 feet?

40. $18\frac{3}{4}\%$ of 480 yards?

41. $33\frac{1}{3}\%$ of \$6.93?

42. $6\frac{2}{3}\%$ of \$4.50?

43. $\frac{3}{8}\%$ of 240 pints?

44. 100% of 48 rods?

45. 200% of \$9.50?

46. 110% of \$6.50?

47. An army of 8450 men lost 22% of its men in battle: how many men did it lose?

48. A man has an income of \$2540, and his expenses are $62\frac{1}{2}\%$ of his income: how much are his expenses?

49. A man having 285 acres of land gave $33\frac{1}{3}\%$ of it to his daughter: how many acres did she receive? How many acres had he left?

50. A drover bought 245 sheep of A, and 60% as many sheep of B: how many sheep did he buy of B?

51. A school enrolled 160 pupils; 55% of them were girls: how many girls were enrolled?

52. A farmer raised in one season 650 bushels of grain, and 64% of it was corn: how many bushels of corn did he raise?

ART. 138. To find a given per cent of any number:

Rule.—*Multiply the number by the given rate per cent, expressed decimally.*

NOTE.—When the rate is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of 100, the per cent may be found by taking $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of the number. Thus, $33\frac{1}{3}\%$ of \$48 is $\frac{1}{3}$ of \$48; $12\frac{1}{2}\%$ of 320 is $\frac{1}{8}$ of 320.

LESSON LII.

TO FIND WHAT PER CENT ONE NUMBER IS OF ANOTHER.

ORAL EXERCISES.

1. What per cent of 12 is 3?

SOLUTION.—1 is $\frac{1}{12}$ of 12, and 3 is $\frac{3}{12}$, or $\frac{1}{4}$ of 12; $\frac{1}{4}$ is $\frac{25}{100}$, or 25%: 3 is 25% of 12.

2. What per cent of 10 is 1? 2? 3? 4? 5? 7?
3. What per cent of 20 is 1? 2? 3? 4? 5? 8?
4. What per cent of 25 is 1? 5? 6? 7? 10? 15?
5. What per cent of 40 is 4? 8? 10? 20? 30?
6. What per cent of 80 is 8? 4? 20? 40? 16?
7. What per cent of 90 is 9? 75 is 25? 200 is 25?

To TEACHERS.—Show that the rate per cent in each of the above problems may also be found by dividing the number which is the percentage by the other number. Thus, (1) $\frac{3}{12} = 3 \div 12 = .25 = 25\%$. Let the pupils solve the 6th and 7th examples on the slate or board.

WRITTEN EXERCISES.

8. What per cent of \$520 is \$23.40?

What per cent of:

9. 320 is 32?
10. 180 is 45?
11. 75 is 13.5?
12. 640 is 48?

PROCESS.

\$520)\$23.40(.045 = $4\frac{1}{2}\%$, *Ans.*
20 80
2600
2600

What per cent of:

13. 84 is 2.1?
14. 10.8 is 1.2?
15. 75 is 4.5?
16. 142.6 is 7.13?
17. 150 is 60?
18. 95 is 14.25?
19. \$650 is \$32.50?
20. 38 lb. is 5.32 lb.?
21. 900 yd. is 112.5 yd.?
22. \$128 is \$5.76?
23. \$7.20 is \$1.08?
24. \$324 is \$356.40?
25. \$324 is \$32.40?
26. \$3.20 is \$3.36?
27. \$2.40 is \$2.04?
28. 12½ cts. is 5 cts.?
29. 12½ cts. is 10 cts.?
30. 25 cts. is 6¼ cts.?
31. 55 cwt. is 4.4 cwt.?
32. 7.5 lb. is .6 lb.?

33. A drover bought 45 horses, and sold 18 of them: what per cent of the drove did he sell?

34. A merchant bought 432 yards of silk, and sold 72 yards: what per cent of the silk did he sell?

35. A school enrolled 225 pupils in a term, and the average number in daily attendance was 198: what was the per cent of attendance?

36. An army of 15450 men lost 1236 men in battle: what per cent of the army was lost?

ART. 139. To find what per cent one number is of another:

Rule.—*Divide the number which is the percentage by the other number, and the quotient, expressed in hundredths, will be the rate per cent.*

LESSON LIII.

TO FIND A NUMBER WHEN A PER CENT OF IT IS GIVEN.

ORAL EXERCISES.

1. 60 is 20% of what number?

SOLUTION.—If 60 is 20% of a number, 1% of it is $\frac{1}{20}$ of 60, which is 3, and 100%, or the number, is 100 times 3, which is *300: 60 is 20%* of 300.

2. 50 is 25% of what number? 10%? 5%? 20%?
3. 75 is 15% of what number? 25%? 5%? 10%?
4. $36 is 3% of how many dollars? 6%? 9%?
5. $70 is 10% of how many dollars? 7%? 35%?
6. 240 acres are 12% of how many acres? 24%?
7. 250 miles are 50% of how many miles? 10%?
8. 500 rods are 25% of how many rods? 125%?

To TEACHERS.—Show that the result obtained above by analysis may also be obtained by dividing the given number by the rate per cent expressed decimally. Thus, (1) $60 \div 20 \times 100 = 60 \div .20 = 300$; (8) 500 rd. $\div 125 \times 100 = 500$ rd. $\div 1.25 = 400$ rd. Let the pupils solve the 5th, 6th, 7th, and 8th examples on the slate or board.

WRITTEN EXERCISES.

9. 3175 is $12\frac{1}{2}\%$ of what number?

PROCESS: $3175 \div .125 = 25400$, *Ans.*

10. 75 is $37\frac{1}{2}\%$ of what number?
11. $23.10 is 5% of what sum of money?
12. $61.60 is 110% of what sum of money?
13. $8.16 is $12\frac{1}{2}\%$ of what sum of money?
14. $180 is $83\frac{1}{3}\%$ of what sum of money?
15. 7.13 is 23% of what number?
16. 2.31 is $33\frac{1}{3}\%$ of what number?
17. 4.2 bushels are $37\frac{1}{2}\%$ of how many bushels?
18. 14.25 tons are 15% of how many tons?
19. $36.50 is $12\frac{1}{2}\%$ of what sum of money?
20. $2.91 is 3% of what sum of money?
21. The number of girls enrolled in a school is 28, which is 40% of the whole number of pupils: how many pupils are enrolled?

ART. 140. To find a number when a per cent of it is given:

Rule.—*Divide the number which is the percentage by the given rate per cent expressed decimally.*

LESSON LIV.

APPLICATIONS OF PERCENTAGE.

ORAL PROBLEMS.

1. A farmer raised 200 bushels of grain, and 20% of it was wheat: how many bushels of wheat did he raise?

2. There are 300 pupils in a graded school, and 60% of them study arithmetic: how many pupils study arithmetic?

3. A pupil wrote 40 words, and misspelled 10% of them: how many words did he misspell?

4. Susan wrote 50 words, and misspelled 2 of them: what per cent did she misspell? What per cent did she spell correctly?

5. A school enrolls 60 pupils, and on a certain day 3 of them were absent: what per cent of the pupils were absent? What per cent were present?

6. A school enrolls 80 pupils, and on a certain day 5% of them were absent: how many pupils were absent?

7. A man undertook a journey of 200 miles, and walked 8% of the distance the first day, 10% of it the second day, and 15% on the third day: how many miles did he walk on each of the first three days?

8. A man, having to make a journey of 200 miles, traveled 40 miles each day: what per cent of the journey did he make each day?

9. A man walked 16 miles, which was 8% of his journey: how many miles in his journey?

10. A regiment contained 400 men, and 50 of them were killed in battle: what per cent of the men were killed?

11. A man paid \$80 for a horse, and sold it for 10% more than it cost him: for how much did he sell it?

SOLUTION.—10% of \$80 is \$8, and \$80 plus \$8 is \$88: he sold *the horse for \$88.*

12. A man bought a horse for \$80, and sold it for \$88: what per cent did he gain?

SOLUTION.—If he gave \$80, and sold for \$88, he gained \$88 less \$80, which is \$8; \$8 is $\frac{8}{80}$, or $\frac{1}{10}$ of the cost, and $\frac{1}{10}$ equals $\frac{10}{100}$, or 10%: he gained 10 per cent.

13. A man bought a buggy for \$75, and sold it for \$90: what per cent did he gain?

14. A dealer paid \$5 for a hat, and sold it at 20% profit: what was the selling price?

15. A dealer paid \$5 for a hat, and sold it for \$6: what per cent did he gain?

16. A dealer sold a hat for \$6, and gained \$1: what per cent did he gain?

17. For how much must silk that cost \$1.20 a yard be sold to gain 25%?

18. Silk that cost \$1.20 a yard was sold for \$1.50: what was the gain per cent?

19. Silk was sold for \$1.50 a yard, at a gain of 30 cts. a yard: what was the per cent of gain?

20. For how much must butter that cost 20 cents a pound be sold to gain 10%? 20%?

21. A carriage that cost \$150 was sold for \$120: what was the per cent of loss?

22. A merchant bought velvet at \$4 a yard, and sold it at \$5 a yard: what per cent did he gain?

23. A merchant bought velvet at \$5 a yard, and sold it at \$4: what was the per cent of loss?

24. A merchant bought velvet at \$4 a yard, and sold it at a gain of 20%: what was the selling price?

WRITTEN PROBLEMS.

25. A school enrolled 320 pupils in a term, and the average number in daily attendance was 82½% of the number enrolled: what was the average daily attendance?

26. A farmer raised 2450 bushels of grain, and 42% of it was wheat, 24% oats, and the rest corn: how many bushels of each kind of grain did he raise?

27. A ship is valued at $15800, and the cargo is worth 15% less than the ship: what is the value of the cargo?

28. A farmer sold 420 pounds of wool, which was 15% of his clip: how much wool did he shear?

29. A man invested $4050, which was 7½% of his property: what was the value of his property?

30. A lady paid $31.50 for a chain, which was 45% of what she paid for a watch: what was the cost of the watch?

31. A man paid $225 a year for a house, which was 15% of his income: what was his income?

32. If 3740 pounds of ore contain 2618 pounds of iron, what per cent of the ore is iron?

33. A man bought a lot for $500, and paid $425: what per cent of the cost did he then owe?

34. An army consisting of 7500 men lost 875 men in battle and 625 men by sickness: what per cent of the men were left?

35. The population of a city in 1870 was 40500, and in 1880 it was 52650: what was the per cent of increase in ten years?

36. A man paid $8750 for a farm, and sold it at 40% profit: what was the selling price?

37. A cargo of wheat which cost $24650 was sold at a loss of 12%: for how much was it sold?

38. A drover paid $135 a head for horses, and sold them at 30% profit: what was the selling price?

39. A house that cost $3840 was sold for $4128: what was the gain per cent?

40. For how much must teas that cost $.90, $1.05, and $1.10 a pound be sold, respectively, to gain 20%? *To gain 10%?* To gain 25%?

41. An agent sold a farm for \$3500, and received a commission of 5%: how much did he receive?

NOTE.—The teacher should explain the business terms used in these problems, and add such information as will be of practical value to the pupil.

42. An attorney collected a debt of \$324.50, and charged 10% for his services: what was his commission?

43. An architect furnished plans and superintended the erection of a building for $2\frac{1}{2}\%$ of its cost, which was \$25400: how much did he receive?

44. A commission merchant sold 320 barrels of flour at \$6.50 a barrel, and charged $4\frac{1}{2}\%$ commission: how much did he receive?

45. A broker bought \$12300 worth of cotton, and charged $\frac{1}{8}\%$: what was his commission?

46. A store worth \$25600 is insured for $\frac{3}{4}$ of its value, at $\frac{1}{2}\%$: what is the premium?

PROCESS: $\begin{cases} \frac{3}{4} \text{ of } \$25600 = \$19200, \text{ amount insured.} \\ \$19200 \times .005 = \$96, \text{ premium.} \end{cases}$

47. A house worth \$7500 is insured for $\frac{2}{3}$ of its value, at $\frac{3}{8}\%$: what is the premium?

48. A man's property is listed at \$10450, and the tax levy is 15 mills on the dollar: what is his tax?

PROCESS: $\$10450 \times .015 = \$156.75$, tax.

49. A man pays a tax of $12\frac{1}{2}$ mills on property listed at \$4960: what is his tax?

50. A merchant imported a lot of silk invoiced at \$32600: what was the duty, at $37\frac{1}{2}\%$?

51. A merchant deducted for cash 5% from a bill of goods amounting to \$540: how much did he receive for the goods?

52. A man asked \$12500 for a farm, but sold it for 6% *off for cash* down: what did he receive for the farm?

INTEREST.

LESSON LV.

ORAL EXERCISES.

1. Mr. Jones borrowed \$100 of a neighbor, and paid him \$6 for the use of the money for one year: what per cent of the \$100 did he pay for its use?

2. If Mr. Jones had paid \$7 for the use of \$100 for one year, what per cent of the money would he have paid?

3. What per cent of \$100 would he have paid if he had paid \$5? \$4? \$8? \$10?

4. If Mr. Jones had paid 6% of the \$100 for the use of it for one year, how much money would he have paid?

5. How much would he have paid if he had paid 8%? 7%? 4½%? 10%? 5%? 8%? 12%?

ART. 141. The money paid for the use of money is called *interest*. The \$6 paid by Mr. Jones for the use of \$100 for one year was *the interest of \$100 at 6 per cent.*

ART. 142. The money for the use of which interest is paid is called the *principal*. The \$100 borrowed by Mr. Jones was the principal, for the use of which he paid \$6 interest.

6. What is the interest of \$100 for 1 year at 6%? At 7%? At 8%? At 5½%? At 10%?

7. What is the interest of \$200 for 1 year at 6%? Of \$300? \$400? \$500? \$800?

SOLUTION.—If the interest of \$100 1 year at 6% is \$6, the interest *of \$200 is twice* \$6, which is \$12; the interest of \$300 is 3 times \$6, *which is \$18;* and so on.

8. What is the interest of \$100 for 2 years at 6%? For 3 years? 4 years? 5 years? 10 years?

SOLUTION.—If the interest of \$100 for 1 year at 6% is \$6, the interest for 2 years is twice \$6, which is \$12.

9. What is the interest of \$50 for 2 years at 6%? For 3 years? $4\frac{1}{2}$ years? 6 years? 8 years? 10 years?

SOLUTION.—The interest of \$50 for 1 year at 6% is $\frac{6}{100}$ of \$50, which is \$3; and the interest for 2 years is twice \$3, which is \$6.

10. What is the interest of \$50 for 1 year at 5%? At 7%? At 8%? At $4\frac{1}{2}$%? At 10%?

11. What part of a year is 1 month? 2 months? 3 months? 4 months? 5 months? 6 months? 8 months? 9 months? 10 months?

WRITTEN EXERCISES.

12. What is the interest of \$80 for 3 years at 6%? For 3 yr. 6 mo.?

PROCESS.

3 yr. 6 mo. = $3\frac{1}{2}$ yr.

\$80
.06
\$4.80, *Int. for* 1 *yr.*
$3\frac{1}{2}$
14.40, *Int. for* 3 *yr.*
2.40, *Int. for* $\frac{1}{2}$ *yr.*
\$26.80, *Int. for* $3\frac{1}{2}$ *yr.*

What is the interest of:

13. \$150 for 2 yr. at 5%?
14. \$85 for 3 yr. at 6%?
15. \$65 for 4 yr. at 8%?
16. \$80 for 2 yr. at 7%?
17. \$85.45 for 2 yr. at 6%?
18. \$300.50 for 5 yr. at 10%?
19. \$420 for 6 mo. at $5\frac{1}{2}$%?
20. \$96 for 1 yr. 6 mo. at 4%?
21. \$164 for 2 yr. 3 mo. at 8%? At 6%?
22. \$120 for 3 yr. 4 mo. at 5%? At 10%?
23. \$84 for 1 yr. 10 mo. at 10%? At 6%?
24. \$800 for 1 yr. 4 mo. at 6%? At 8%?
25. \$124 for 2 yr. 7 mo. at 6%? At 12%?

26. \$4.50 for 1 yr. 8 mo. at 8%? At 7½%?
27. \$3.50 for 3 yr. 6 mo. at 4%? At 8%?
28. \$126 for 1 yr. 4 mo. at 6%? At 10%?
29. \$240 for 1 mo. at 12%? At 5%? At 8%?

DEFINITIONS.

ART. 143. **Interest** is money paid for the use of money.

The **Principal** is the money for the use of which interest is paid.

The **Amount** is the sum of principal and interest.

The **Rate of Interest** is the number of hundredths of the principal paid for its use for one year.

NOTE.—Rate of interest means either the rate per cent of the principal or the rate simply. Thus, it may be said that the legal rate of interest is 6 per cent, or 6.

LESSON LVI.

THE SIX PER CENT METHOD.

ORAL EXERCISES.

1. What is the interest of \$1 for 1 year at 6%? For 2 yr.? 3 yr.? 4 yr.? 5 yr.? 10 yr.?

SUGGESTION.—The interest of \$1 for 1 yr. at 6% is $\frac{6}{100}$ of \$1, which is 6 cts.; the interest for 2 years is 2 times 6 cts., which is 12 cts.; and so on.

2. What is the interest of \$1 for 1 month at 6%? For 2 mo.? 4 mo.? 6 mo.? 8 mo.? 10 mo.?

SOLUTION.—The interest of \$1 for 1 mo. at 6% is $\frac{1}{12}$ of 6 cts., or ½ of 1 cent, which is 5 mills, and for 2 months the interest is twice 5 mills, which is 10 mills; and so on.

3. What is the interest of \$1 for 3 months at 6%? *For 5 mo.?* 7 mo.? 9 mo.? 11 mo.?

4. What is the interest of \$1 for 1 yr. 2 mo. at 6%? 1 yr. 4 mo.? 2 yr. 6 mo.? 2 yr. 8 mo.? 1 yr. 10 mo.?

5. What is the interest of \$1 for 1 day at 6%? For 2 days? 3 days? 4 days? 6 days?

SOLUTION.—The interest of \$1 for 1 day at 6% is $\frac{1}{60}$ of 5 mills, which is $\frac{1}{12}$ mill, and the interest for 2 days is $\frac{2}{12}$ mill; for 3 days, $\frac{3}{12}$ mill; for 4 days, $\frac{4}{12}$ mill; for 6 days, $\frac{6}{12}$ mill, or 1 *mill.*

6. What is the interest of \$1 for 12 days at 6%? 18 days? 24 days? 7 days? 9 days?

SUGGESTION.—The interest of \$1 for 1 day is $\frac{1}{12}$ mill, and for 6 days 1 mill.

7. What is the interest of \$1 for 13 days at 6%? 15 days? 21 days? 20 days? 14 days?

8. What is the interest of \$1 at 6% for 1 mo. 6 da.? 2 mo. 12 da.? 4 mo. 15 da.? 2 mo. 20 da.?

9. For 4 mo. 24 da.? 6 mo. 18 da.? 6 mo. 6 da.? 4 mo. 13 da.? 2 mo. 21 da.?

10. For 1 yr. 2 mo. 12 da.? 2 yr. 2 mo. 24 da.? 1 yr. 4 mo. 6 da.? 3 yr. 6 mo. 14 da.?

WRITTEN EXERCISES.

11. What is the interest of \$1 at 6% for 2 yr. 3 mo. 14 da.?

PROCESS.

Int. of \$1 for 2 yr. = 6 c. × 2 = \$.12
" " " " 3 mo. = 5 m. × 3 = .015
" " " "14 da. = $\frac{1}{6}$ m. × 14 = .002$\frac{1}{3}$
" " " " 2 yr. 3 mo. 14 da. = \$.137$\frac{1}{3}$

OR:

Int. for 2 yr. = \$.12
" " 3 mo. = .015
" "14 da. = .002$\frac{1}{3}$
\$.137$\frac{1}{3}$

What is the interest of \$1, at 6%, for:

12. 1 yr. 9 mo. 6 da.?
13. 2 yr. 1 mo. 25 da.?
14. 3 yr. 3 mo. 20 da.?
15. 1 yr. 1 mo. 24 da.?
16. 2 yr. 2 mo. 2 da.?
17. 1 yr. 1 mo. 1 da.?
18. 7 mo. 7 da.?
19. 5 yr. 5 da.?

20. What decimal part of \$1 is \$.07? \$.045? \$.08? \$.12½? \$.24⅜? \$.08⅜? \$.07⅜? \$.13⅞?

ANSWER.—\$.07 = .07 of \$1; \$.045 = .045 of \$1; and so on.

21. In the above examples, from the 12th to the 19th inclusive, what decimal part of \$1 is the interest?

22. What is the interest of \$425 for 2 yr. 3 mo. 12 da., at 6%?

PROCESS.

	\$425	
Int. of \$1 for:	.137	
2 yr. = \$.12	2975	
3 mo. = .015	1275	
12 da. = .002	425	
\$.137	\$58.225, *Int.*	

Since the interest of \$1 for 2 yr. 2 mo. 12 da. at 6% is \$.137, the interest of \$425 at 6% for the same time is 425 times \$.137, or .137 times \$425, which is \$58.225.

NOTE.—The teacher should show that \$.137×425 = \$425×.137, and why the latter method is preferable to the former in computing interest.

23. What is the interest of \$145.60 for 1 yr. 5 mo. 24 da., at 6%? For 2 yr. 2 mo. 9 da.? 2 yr. 24 da.?

What is the interest at 6% of:

24. \$64.20 for 1 yr. 3 mo.? For 5 mo. 15 da.?

25. \$85.50 for 1 yr. 1 mo. 1 da.? For 4 yr. 10 da.?

26. \$184.80 for 9 mo. 27 da.? For 2 yr. 25 da.?

27. \$300.50 for 3 yr. 8 mo.? For 1 yr. 7 mo.?

28. \$205.25 for 1 yr. 4 mo. 15 da.? For 3 yr. 5 mo.?

29. \$500 for 1 yr. 1 mo. 18 da.? For 4 mo. 15 da.?

30. \$75.60 for 3 yr. 5 mo. 12 da.? For 3 yr. 15 da.?

31. \$450 for 7 mo. 20 da.? For 2 yr. 2 mo. 2 da.?

32. \$60.45 for 1 yr. 7 mo.? For 3 yr. 3 mo. 3 da.?

33. \$87.60 for 9 mo. 15 da.? For 1 yr. 9 mo. 15 da.?

34. \$19.45 for 6 mo. 6 da.? For 2 yr. 6 mo. 14 da.?

35. \$10.60 for 5 mo. 5 da.? For 2 yr. 5 mo. 5 da.?

36. *\$200* for 1 yr. 1 mo. 1 da.? For 7 mo. 15 da.?

37. What is the interest of \$31.20 from Oct. 23, 1875, to Apr. 12, 1879, at 8%?

PROCESS.

1879	4	12	
1875	10	23	
3 yr.	5 mo.	19 da.	

Int. of \$1 at 6% for:

3 yr. = \$.18
5 mo. = .025
19 da. = .003⅙
\$.208⅓

\$31.20
.208⅓
520
24960
6240
3)\$6.49480, *Int. at 6%.*
2.16493, " " *2%.*
\$8.65973, " " *8%.*

What is the interest of:

38. \$540 from Feb. 5, 1879, to Mar. 15, 1882, at 7%?

39. \$47.60 from July 5, 1880, to Feb. 9, 1882, at 9%?

40. \$160.50 from Aug. 5, 1879, to Mar. 17, 1881, at 8%?

41. \$12.50 from Nov. 15, 1880, to Apr. 3, 1882, at 5%?

42. \$533.80 from Feb. 20 to Dec 5, at 5%?

43. \$54.60 from Mar. 5 to Sep. 21, at 4%?

44. \$758.50 from Oct. 20, 1880, to May 12, 1882, at 10%?

45. \$235.25 from May 12, 1881, to Apr. 1, 1882, at 6%?

What is the amount of:

46. \$240 from Feb. 15, 1878, to Apr. 27, 1881, at 8%?

47. \$180 from May 14, 1880, to Apr. 3, 1882, at 7%?

48. \$137.50 from July 3, 1880, to Feb. 6, 1882, at 5%?

49. \$153.80 from Oct. 25, 1880, to Jan. 1, 1882, at 6%?

50. \$210.62 from Apr. 4 to Nov. 20, at 4%?

51. A man borrowed \$460, July 28, 1866, and paid it May 16, 1869, with interest at 5%: what was the amount?

52. A note of \$243.75, dated June 8, 1873, was paid Nov. 14, 1875, with interest at 10%: what was the amount?

ART. 144. 1. To compute interest at 6 per cent:

Rule.—*Find the interest of $1 for the given time, and then multiply the given principal by the abstract decimal which corresponds to the interest of $1.*

NOTE.—The interest of $1 may be found by taking six times as many cents as there are years, one half as many cents as there are months, and one sixth as many mills as there are days.

2. To compute interest at any rate per cent:

Rule.—*Find the interest at 6 per cent, and then increase or diminish this interest by such a part of itself as will give the interest at the given rate.*

NOTE.—The following table denotes the part of the interest to be added or subtracted to give the interest at the given per cent:

$7\% = 6\% + \frac{1}{6}$ of 6%	$10\% = \frac{1}{6}$ of $6\% \times 10$
$7\frac{1}{2}\% = 6\% + \frac{1}{4}$ of 6%	$5\% = 6\% - \frac{1}{6}$ of 6%
$8\% = 6\% + \frac{1}{3}$ of 6%	$4\% = 6\% - \frac{1}{3}$ of 6%
$9\% = 6\% + \frac{1}{2}$ of 6%	$4\frac{1}{2}\% = 6\% - \frac{1}{4}$ of 6%

LESSON LVII.

BANK DISCOUNT.

ART. 145. A **Promissory Note** is a written agreement by one party to pay another a specified sum of money at a specified time.

The sum of money specified is called the *Face* of the note.

The person who signs a note is its *Maker;* the person to whom it is payable is the *Payee;* and the owner is the *Holder.* The person who writes his name on the back of a note as security for its payment, is an *Indorser.*

ART. 146. The following is the common form of a *time note:*

$65\frac{50}{100}$ Lafayette, Ind., Jan. 10, 1883.

Three months after date I promise to pay to John Wilson, or order, Sixty-five $\frac{50}{100}$ dollars, with interest at 8 per cent, for value received.

Henry Smith.

The face of the above note is \$65.50, Henry Smith is its maker, and John Wilson is the payee. If John Wilson wishes to transfer this note to David King, he should write on the back of it "Pay to order of David King," and sign his name. John Wilson would then be the *indorser* and David King the *holder*.

ART. 147. When a bank loans money, the borrower gives his note, payable at a specified time *without interest*. The interest on this note for the time, plus three days, is subtracted from its face, and the remainder, called the **proceeds**, is paid to the borrower.

The interest thus deducted is called **discount**, and the three days added to the time are called *days of grace*.

A note is *payable* at a bank at the time mentioned in it, and is *due* on the third day following.

WRITTEN PROBLEMS.

1. A bank discounted a note of \$250, payable in 60 days, at 8%: what were the proceeds?

		PROCESS.
\$250		60 days + 3 days = 63 days.
.0105		Int. of \$1 at 6% for 63 da.=\$.0105
1250		
250		
3)\$2.6250,	*Dis. at 6%.*	\$250.00
8750		3.50
\$3.50,	*Dis. at 8%.*	\$246.50, Proceeds.

What are the bank proceeds of a note of:

2. $22.50, payable in 60 days, discounted at 6%?

3. $720, payable in 30 days, discounted at 8%?

4. $62.40, payable in 45 days, discounted at 10%?

5. $125, payable in 90 days, discounted at 7½%?

6. What are the proceeds of a note of $90.60, dated March 10, 1876, and payable June 5, 1876, discounted by a bank at 9%?

PROCESS.

In March, 21 days	$90.60
" April, 30 "	.015
" May, 31 "	45300
" June, 5 "	9060
Grace, 3 "	2)$1.35900
6)90 days	6795
Int. of $1 at 6% = $.015	$2.0385, *Dis. at 9%.*

$90.60 − $2.0385 = $87.5615, *Proceeds.*

NOTE.—When the time of a note is short, it is the general custom of bankers to compute interest for the actual number of days in the time, including grace, each day being considered as $\frac{1}{360}$ of a year. This mode of finding the time of interest is called the *Method by Days.*

7. What are the proceeds of a note of $240.60, dated July 2, and payable Sept. 30, discounted by a bank at 6%?

8. What are the proceeds of a note of $142, dated Aug. 15, and payable Nov. 4, discounted at 10%?

9. A note of $360, dated Sept. 15, and payable Nov. 15, was discounted by a bank at 8%: what were the proceeds?

10. A man sold a note of $150, due in 6 months, at a discount of 10% for the time, without grace: how *much did he* receive for the note?

11. A bank discounted a note of $225, payable in 60 days, without interest, at 8%: what were the proceeds?

12. A note for $650, payable Dec. 10th, without interest, was discounted Sep. 15th, at 7%: what were the proceeds?

ART. 148. **Bank Discount** is interest paid *in advance.* It is also called *Bank Interest.*

The **Proceeds** of a note is the difference between its face and its interest.

ART. 149. To compute bank discount:

Rule.—1. *Find the interest of the sum discounted for the number of days in the time plus three days.*

2. *Subtract the discount from the sum discounted, and the difference will be the proceeds.*

NOTES.—1. When the time of a note is expressed in *months*, the interest is computed for the number of *calendar months* in days, plus three days. See problem 6.

2. The following method of computing interest at 6%, by days, is also used by business men:

Remove the decimal point in the sum of money three places to the left, and multiply the result by $\frac{1}{6}$ of the number of days; or multiply the result by the number of days, and divide by 6.

LESSON LVIII.

DRAFTS, BONDS, AND STOCKS.

ART. 150. A **Draft** is an order by one person upon another to pay a specified sum to a third person named. It is also called a *Bill of Exchange.*

The process of making payments at distant places by the *remittance of drafts* is called *Exchange.*

ART. 151. **Bonds** are interest-bearing notes issued by nations, states, cities, railroad companies, and other corporations, as a means of borrowing money.

The market value of bonds is quoted at a certain per cent of their face, or par value. Bonds quoted at 109 are worth, in currency, 109% of their face.

Capital invested in business is divided into equal parts, called *Shares*. The bonds issued by companies and the shares of their capital are both called *Stocks*.

TO TEACHERS.—For fuller information respecting drafts, bonds, and stocks see "New Complete Arithmetic"

WRITTEN PROBLEMS.

1. What will be the cost of a draft on New York for $640, when exchange is $\frac{3}{8}\%$ premium?

PROCESS: $\begin{cases} \$640 \times .00\frac{3}{8} = \$2.40, \textit{ Cost of exchange.} \\ \$640 + \$2.40 = \$642.40, \textit{ Cost of draft.} \end{cases}$

2. What is the cost of a draft on Cincinnati for $275, at $\frac{1}{4}\%$ discount? At $\frac{1}{8}\%$ premium?

3. What is the cost of a draft on Chicago for $450, at $\frac{1}{8}\%$ discount? At $\frac{1}{4}\%$ premium?

4. What is the cost of a draft on Louisville for $225, exchange being at $\frac{1}{5}\%$ premium?

5. What is the cost of a draft on Philadelphia for $1575, exchange being $\frac{3}{4}\%$ premium?

6. What is the cost of a draft on St. Louis for $350, exchange being at $\frac{1}{5}\%$ premium?

7. What is the cost of a draft on Boston for $2120 at $\frac{5}{8}\%$ discount? At $\frac{3}{4}\%$ premium?

8. What is the cost of a draft on New York for $265.50, at $\frac{3}{8}\%$ discount? At $\frac{1}{4}\%$ discount?

9. What is the cost of a draft on Baltimore for $280.60, at $\frac{3}{4}\%$ discount? At $\frac{1}{2}\%$ discount?

10. A merchant in St. Louis wishes to remit $2450 to a creditor by draft on New York: what will be the cost of the draft, at $\frac{3}{4}\%$ premium?

11. A merchant in New Orleans bought a draft on New York for $5600, at $\frac{5}{8}\%$ discount: what was the cost of the draft?

12. When United States bonds (5's) are quoted at $112\frac{1}{2}$, what will be the cost of $1500 in bonds?

PROCESS: $\$1500 \times 1.125 = \$1687.50$, *Ans.*

NOTE.—When bonds or stocks are quoted at $112\frac{1}{2}$, they are worth $112\frac{1}{2}\%$ of their face, that is, are selling at $12\frac{1}{2}\%$ premium. When bonds or stocks are quoted at 90, they are worth 90% of their face, that is, are selling at 10% discount.

13. What is the cost of three $1000 United States bonds, $4\frac{1}{2}$'s, quoted at $107\frac{1}{2}$?

14. What will be the cost of 40 shares ($1000 each) of New York Central stock quoted at $133\frac{1}{3}$?

15. What will be the cost of 50 shares ($1000 each) of Central Pacific at $89\frac{1}{4}$?

16. What will be the cost of 60 shares ($1000 each) of Union Pacific at $113\frac{1}{4}$?

LESSON LIX.

REVIEW PROBLEMS.

1. A grain dealer bought a car load of wheat for $498, and sold it at a loss of 8%: how much did he receive for the wheat?

2. A lady sold her horse and phaeton for $324, and thereby lost 20%: what did they cost her?

3. A merchant bought cloth at $2.52 a yard: for how much must he sell it to gain 16%? 20%?

4. At what prices must muslins that cost 7 cts., 9 cts., and 10 cts. a yard, be sold to gain 25%? 20%?

5. A farmer bought a cow for $42, and sold her for $58: what per cent profit did he make?

6. If oranges sold at 45 cts. a dozen give a profit of 25%, what would be the per cent of profit if sold at 40 cts.? At 50 cts.?

7. A commission-merchant charged $34.56 for selling 1280 bushels of potatoes at 60 cts. a bushel: what per cent commission did he receive?

8. If I ship a commission-merchant 440 barrels of apples, which he sells at $3.25 per barrel, at 5% commission, how much money should he remit to me?

9. The annual premium on a life policy for $5800 is $174: what is the rate per cent?

10. A man paid a premium of $177.66 for insuring a building at $\frac{3}{4}$%: what was the amount of insurance?

11. A city insures its public library building, worth $30,000, and the books, valued at $14,700, for $\frac{2}{3}$ of their value, at $1\frac{1}{2}$%: what annual premium does it pay?

12. What will be the cost of a draft on New York for $85, at $\frac{3}{5}$% premium?

13. A note for $741.50, dated Feb. 20, 1880, and bearing interest at $5\frac{1}{2}$%, was paid June 25, 1882: what was the amount?

14. A note of $280.60, dated July 1, 1881, was paid May 10, 1882, with interest at 8%: what was the amount?

15. A note of $474, dated Aug. 15 and payable Nov. 1, was discounted by a bank at 7%: what were the proceeds?

16. INDIANAPOLIS, IND., JAN. 21, 1880.

Two years from date, for value received, I promise to pay James R. Wilson, or order, one hundred and twenty-five and $\frac{25}{100}$ dollars, with interest at 10%?

R. E. CRAIN.

What was the amount of this note Dec. 21, 1882?

ANSWERS TO WRITTEN PROBLEMS.

N. B.—The answers to the examples in Part I are not given, and, in the succeeding parts, when a problem has two or more answers, with few exceptions, only the last answer is given. *See Note to Teachers, page 268.*

PART II.

ADDITION.

Page 77.

22. 921566.
23. 270724.
24. 270527.
25. 307352.
26. 214353.
27. 307777.
28. 281979.
29. 2429123.
30. 2726863.
31. 3178699.
32. 435530.
33. 57154930.
34. 553002.
35. 24757307.
36. 30787140.

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37. $305188.
38. $1144628.
39. $7948026.
40. $3020409.
41. $526290.
42. $1095967.
43. $209200.
44. 4444844.
45. 395096.
46. 53293685.
47. 22057045.

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48. 157973346.

Page 81.

24. 184 days.
25. 181 days.
26. 8996 pounds.
27. 15084 bushels.
28. 1949 voters.
29. 12326 youths.
30. 649 acres.
31. 117557 pounds

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32. 1462 acres.
33. 1064 miles.
34. 1037 miles.
35. $10585.
36. $2500.
37. 192975 sq. mi.
38. 66465 sq. mi.
39. 19788376.
40. 11084613.

SUBTRACTION.

Page 85.

21. 1141.

Page 86.

23. 127.
24. 1125.
25. 109.
26. 2297.
27. 2909.
28. 404.
29. 7061.
30. 3314.
31. 9825.
32. 59019.
33. $1228.
34. $41076.
35. $40955.
36. $23321.
37. 8481.
38. 420135.
39. 39920612.
40. 48998052.

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18. 354 pounds.

Page 88.

19. 183 acres.
20. $2080.
21. 1992 bushels.
22. 478 youths.
23. $485.
24. $10339
25. 156 years.
26. 59 years.
27. 128 years.
28. 6926 feet.
29. 5618 feet.
30. 22772 men.
31. 91895 sq. mi.
32. 11508427.

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11. 108 yards; 42 yards.
12. $2566.
13. 599 men.
14. 6635 bushels; 1185 bushels.
15. $945; $705; $240.
16. 425 bushels.
17. 24290 sq. mi.
18. 2598.
19. 3728.
20. 680134.
21. $1201.

MULTIPLICATION.

Page 92.

12. 3915.
13. 51066.

Page 93.

14. 53664.
15. 72891.
16. *129888.*
17. 1436400.
18. 1024128.
20. 317814.
21. 195468.
22. 2664552.
23. 3445200.
24. 3128040.
25. 3136364.

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26. 5590173.
27. 5481522.
29. 27440670.
30. 13306938.
31. 29014132.
32. 165618574.
33. 142080798.

34. 13095430.
35. 1382950.
36. 5503716.
37. 17347225.
38. 40001688.
39. 92959677.
41. 2079774.
42. 47487540.
43. 24354675.
44. 13867680.
45. 23062642.
46. 30523415.
47. 284212341.
48. 216409884.
49. 89762000.
50. 175044000.

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51. 153712800.
52. 92338300.
54. 288502000.
55. 15323600.
56. 252504000.
57. 260100000.
58. 6256340000.
60. 338400000.
61. 58112000.
62. 966280000.
63. 774400000.
64. 3074000.
65. 154200000.
66. $68875.
67. $3651150.
68. $29264000.
69. $2811200.

Page 97.

33. 8000 rods.
34. 19125 miles.
35. 33075 tons.
36. 9720 miles.
37. 22080 yards.
38. $14180.
39. 1440 minutes; 10080 minutes.
40. 330325 days.
41. 91520 pounds.

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42. 2451960 pounds.
43. $2295125.
44. 3194400 feet.
45. 8160000 miles.
46. 70000 times.
47. 68600 pounds.
48. 268800 sheets.
49. 94240 pounds.
50. 40600 passengers.
51. 604800 seconds
52. 16588800000 miles.
53. 3924000 feet.
54. 11700 pounds.
55. $163200.

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56. $30375.
57. $1080.
58. $4440.
59. $21120.
60. $840.
61. $1920.
62. 36180 men.
63. 19500 miles.
64. 58800 stalks.
65. 364450 pounds
66. 357700 pounds
67. 268800 pounds
68. 15552 cents, or $155.52.
69. 473040 pounds
70. 149600 cents, or $1496.00.

Page 102.

16. 360000.
17. 197600.
18. 570000.
19. 30856.
20. 45067.
21. $50.
22. $684.
23. $288.
24. $330.
25. $420.
26. $900.
27. $1330.

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28. $85.
29. $80000.
30. $340, $3060.

DIVISION.

Page 104.

14. 310230.
15. 202102.
16. 20101.
17. 14144.
18. 2310.
19. 9723.
20. 81030.
21. 66434.

Page 105.

23. $200759\frac{3}{8}$.
24. $522918\frac{8}{9}$.
25. $1518914\frac{5}{7}$.
26. $23093\frac{4}{9}$.
27. $504809\frac{3}{8}$.
28. $45251\frac{8}{12}$.
29. $1145007\frac{7}{9}$.
1. $8671\frac{6}{7}$.

Page 106.

3. 212.
4. 113.
5. 213.
6. 312.
7. 31.
8. 312.
9. 222.
10. 321.
11. 527.
12. $6550\frac{968}{1067}$.
13. *4725.*
14. $3437\frac{675}{1048}$.
15. $4326\frac{865}{1086}$.
16. 433.
18. 13002.
19. 22003.
20. 2202.
21. 25002.
22. 702.
23. 4802.
24. 241.
25. 97.
26. 594.
27. 15196.
28. 6125.
29. $58.
30. $81.
31. $4096.
32. 271.
33. $247\frac{324}{485}$.

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35. $508\frac{15}{48}$.
36. $579\frac{360}{806}$.
37. $496\frac{3919}{5728}$.
38. $119\frac{693}{1806}$.
39. $209\frac{5620}{6785}$.
40. $768\frac{499}{2088}$.
41. 507.
42. $441\frac{144}{608}$.
43. 809.
44. 6007.
46. 28.
47. 456.
48. 1870.
49. $384\frac{50}{1000}$.
50. $23\frac{45}{10000}$.
51. $450\frac{860}{1000}$.

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53. 8.
54. 40030.
55. 935.
57. $119\frac{2284}{3000}$.
58. $18\frac{2780}{5400}$.
59. $234\frac{385}{940}$.
60. 12.
61. $48\frac{3075}{4500}$.
62. 9020.
63. $85\frac{2000}{4500}$.
64. $102\frac{5924}{7400}$.
65. 20.
66. 8.
67. 376.
68. 985.

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21. 213 bushels.
22. 132 hours.
23. 175 days.
24. 93 boxes.
25. 243 barrels.
26. 535 days.
27. 66 yards.
28. 4360 bushels.

29. 760 hogsheads
30. 2430 boxes.
31. 47 farms.
32. 49 days.
33. 2470 years.
34. 36 hours.
35. 1512 weeks.
36. 405 head.

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37. 207 days.
38. 603 days.
39. 640 barrels.
40. 2700 reams.
41. 4800 hours.
42. 480 lots.
43. 44 cars.
44. 40 regiments.

FACTORS, DIVISORS, AND MULTIPLES.

Page 115.

16. 2, 2, 2, 3, 3.
17. 2, 2, 5, 5.
18. 2, 2, 2, 3, 3, 3.
19. 2, 2, 2, 2, 2, 2, 2, 2
20. 2, 2, 2, 5, 7.

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22. 3, 3.
24. 9.
25. 28.
26. 25.
27. 21.
28. 4.
29. 45.
30. 36.
31. 12.
32. 24.
33. 35.
34. 36.
35. 54.
36. 25.
37. 21.
38. 24.
39. 18.
40. 25.
41. 9.

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5. 9.
6. 24.
7. 15.
8. 36.
9. 12.
10. 27.
11. 4.
12. 45.
13. 24.
14. 21.
15. 54.
16. 25.
17. 21.
18. 24.
19. 36.

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9. 60.
10. 168.
11. 480.
12. 108.
13. 240.
14. 144.
15. 480.
16. 300.
17. $250.
18. $144.
19. 216.
20. 128.
21. 180.
22. 96.
23. 300.
24. 252.
25. 72.
26. 60.
27. 120.

Page 122.

26. 406.
27. 42909.
28. 22962.
29. 156.
30. 84.
31. 36.
32. 82532.
33. 2400.

34. \$14.
35. \$285.
36. 10 cows.
37. \$1490; \$2010.
38. \$3125; \$625.
39. 3700 bushels.

Page 123.

40. \$1128.
41. 22750 cents; 3500 cents.
42. \$432.
43. 2700 pass'gers.
44. \$18160.
45. \$1190.
46. 56450 cents, or \$564.50
47. \$1150; \$350.
48. \$55.

FRACTIONS.

Page 129.

17. $\frac{773}{8}$.
18. $\frac{559}{12}$.
19. $\frac{696}{11}$.
20. $\frac{1077}{20}$.
21. $\frac{499}{15}$.
22. $\frac{1199}{14}$.
23. $\frac{551}{16}$.
24. $\frac{629}{18}$.
25. $\frac{1656}{7}$.
26 $\frac{1751}{16}$.
27. $\frac{1509}{20}$.
28. $\frac{1457}{30}$.
29. $\frac{6927}{100}$.
30. $\frac{18609}{200}$.
31. $\frac{8485}{16}$.
32. $\frac{1461}{4}$.
33. $\frac{15307}{100}$.
34. $\frac{4911}{24}$.
35. $\frac{20018}{100}$.
36. $\frac{75041}{1000}$.

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15. $17\frac{3}{12}$.
16. $6\frac{4}{15}$.
17. $8\frac{9}{12}$.
18. $15\frac{7}{20}$.
19. 6.
20. $3\frac{3}{24}$.
21. $10\frac{12}{30}$.
22. 2.
23. 11.
24. $20\frac{2}{16}$.
25. $20\frac{1}{21}$.
26. 10.
27. $12\frac{12}{41}$.
28. $21\frac{9}{35}$.
29. 9.
30. $28\frac{14}{22}$.
31. $17\frac{30}{45}$.
32. $45\frac{8}{20}$.
33. $27\frac{28}{36}$.
34. $80\frac{5}{50}$.

Page 132.

16. $\frac{8}{9}$.
17. $\frac{1}{2}$.
18. $\frac{3}{8}$.
19. $\frac{7}{20}$.
20. $\frac{2}{8}$.
21. $\frac{1}{8}$.
22 $\frac{24}{25}$.
23. $\frac{3}{7}$.
24. $\frac{8}{5}$.
25. $\frac{1}{11}$.
26. $\frac{5}{7}$.
27. $\frac{91}{90}$, or $1\frac{1}{90}$.
28. $\frac{77}{120}$.
29. $\frac{1}{2}$.
30. $\frac{1}{2}$.
31. $\frac{3}{4}$.

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15. $\frac{76}{96}$.
16. $\frac{44}{84}$, $\frac{46}{84}$.
17. $\frac{42}{72}$, $\frac{32}{72}$, $\frac{30}{72}$.
19. $\frac{21}{36}$, $\frac{10}{36}$, $\frac{28}{36}$.
20. $\frac{30}{42}$, $\frac{27}{42}$, $\frac{32}{42}$.
21. $\frac{9}{12}$, $\frac{10}{12}$, $\frac{7}{12}$.
22. $\frac{12}{18}$, $\frac{10}{18}$, $\frac{15}{18}$.
23. $\frac{18}{30}$, $\frac{21}{30}$, $\frac{16}{30}$.
24. $\frac{12}{16}$, $\frac{14}{16}$, $\frac{9}{16}$.

25. $\frac{70}{14}$, $\frac{6}{14}$, $\frac{7}{14}$.
26. $\frac{4}{24}$, $\frac{10}{24}$, $\frac{7}{24}$.
27. $\frac{12}{80}$, $\frac{9}{80}$, $\frac{8}{80}$, $\frac{11}{80}$.
28. $\frac{16}{86}$, $\frac{33}{86}$, $\frac{84}{86}$, $\frac{28}{86}$.
29. $\frac{90}{100}$, $\frac{65}{100}$, $\frac{56}{100}$, $\frac{66}{100}$.

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12. $2\frac{1}{2}$.
13. $2\frac{11}{15}$.
15. $1\frac{9}{10}$.
16. $2\frac{3}{8}$.
17. $1\frac{5}{24}$.
18. $1\frac{17}{30}$.
19. $1\frac{7}{10}$.
20. $1\frac{1}{15}$.
21. $1\frac{23}{24}$.
22. $2\frac{384}{1005}$.
23. $1\frac{1}{4}$.
24. $\frac{2}{5}$.
25. $1\frac{5}{7}$.
26. $2\frac{1}{10}$.
27. $2\frac{7}{12}$.
28. $2\frac{16}{21}$.
29. $2\frac{11}{20}$.
30. $1114\frac{1}{12}$.

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32. $175\frac{7}{8}$.
33. $115\frac{1}{8}$.
34. 315.
35. $551\frac{1}{10}$.
36. $3623\frac{1}{8}$.
37. $1164\frac{11}{21}$.
38. $3528\frac{1}{8}$.
39. $1180\frac{79}{84}$.

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11. $\frac{1}{8}$.
12. $\frac{1}{5}$.
14. $\frac{5}{24}$.
15. $\frac{13}{30}$.
16. $\frac{9}{50}$.
17. $\frac{11}{36}$.
18. $\frac{1}{42}$.
19. $\frac{1}{20}$.
20. $\frac{2}{45}$.
21. $\frac{1}{6}$.
22. $\frac{38}{50}$.
23. $\frac{7}{24}$.
24. $\frac{17}{42}$.
25. $\frac{11}{60}$.
27. $36\frac{5}{12}$.
28. $32\frac{3}{4}$.
29. $49\frac{2}{5}$.
30. $111\frac{1}{24}$.
31. $21\frac{1}{15}$.
32. $18\frac{1}{6}$.
33. $155\frac{7}{12}$.
34. $66\frac{2}{3}$.

Page 139.

1. $\frac{1}{12}$.
2. $\frac{11}{20}$.
3. $\frac{1}{8}$.
4. $\frac{1}{8}$.
5. $\frac{1}{6}$.
6. $\frac{5}{8}$.
7. $\frac{5}{12}$.
8. $\frac{1}{20}$.
9. $\frac{5}{12}$.
10. $1\frac{1}{3}$.
11. $1\frac{1}{6}$.
12. $1\frac{23}{24}$.
13. $\frac{103}{120}$.
14. $\frac{1}{8}$.
15. $\frac{67}{120}$.
16. $2\frac{7}{20}$.
17. $8\frac{3}{8}$.
18. $4\frac{11}{60}$.
19. $\$66\frac{2}{3}$.
20. $\$50\frac{1}{6}$.
21. $91\frac{11}{15}$.
22. $345\frac{1}{4}$.
23. $9\frac{1}{8}$.
24. $51\frac{23}{24}$.
25. $80\frac{5}{24}$.
26. $23\frac{7}{24}$.
27. $\frac{9}{25}$.
28. $19\frac{23}{80}$.

Page 141.

18. $42\frac{4}{9}$.
19. $444\frac{1}{2}$.
20. $399\frac{3}{7}$.
21. $370\frac{7}{11}$.
22. 186.
23. 145.
24. 2110.
25. 1976.

Page 142.

14. $\frac{8}{7}$.
15. $\frac{1}{2}$.
16. $\frac{1}{2}$.
17. $\frac{7}{32}$.
18. $\frac{5}{11}$.
19. $\frac{4}{5}$.
20. $\frac{5}{88}$.
21. $\frac{1}{3}$.
22. $\frac{3}{5}$.
23. $1\frac{7}{8}$.
24. 1.
25. $1\frac{1}{7}$.
26. 3.
27. 1.
28. 1.
29. $5\frac{1}{2}$.
30. 1.
31. $\frac{2}{11}$.

Page 143.

11. $6\frac{1}{4}$.
12. $3\frac{1}{3}$.
13. $8\frac{3}{5}$.
14. $15\frac{3}{5}$.
15. 21.
16. $34\frac{1}{45}$.
17. $19\frac{1}{2}$.
18. $7\frac{1}{2}$.
19. $133\frac{1}{3}$.
20. 225.
21. $2266\frac{2}{3}$.
22. *2085.*

Page 145.

8. 20.
9. $56\frac{7}{8}$.
10. $50\frac{4}{7}$.
11. $69\frac{3}{5}$.
12. $118\frac{6}{11}$.
13. 260.
14. $176\frac{7}{16}$.
15. $256\frac{1}{5}$.
16. $119\frac{1}{25}$.
17. $96\frac{2}{3}$.
18. $247\frac{3}{16}$.
19. $553\frac{3}{88}$.
21. 792.
22. 1350.
23. 2292.
24. $3136\frac{1}{2}$.
25. 5670.
26. $13533\frac{1}{3}$.
27. $6094\frac{2}{7}$.
28. $38042\frac{2}{5}$.
29. 45600.

Page 146.

8. $\frac{1}{4}$.
9. $\frac{2}{3}$.
10. $\frac{1}{6}$.
11. $\frac{5}{18}$.
12. $\frac{49}{120}$.
13. $\frac{1}{2}$.
14. $\frac{2}{3}$.
15. $\frac{1}{4}$.
16. $1\frac{1}{6}$.
17. $2\frac{1}{12}$.
18. $6\frac{1}{4}$.
19. $11\frac{1}{3}$.
20. $15\frac{1}{8}$.
21. $15\frac{5}{8}$.
22. $39\frac{1}{16}$

Page 147

11. $\frac{1}{11}$.
12. $\frac{1}{14}$.
13. $\frac{7}{120}$.
14. $\frac{4}{39}$.
15. $\frac{2}{15}$.
16. $\frac{2}{17}$.
17. $\frac{4}{21}$.
18. $\frac{7}{25}$.
19. $4\frac{1}{18}$.
20. $4\frac{8}{85}$.
21. $9\frac{7}{20}$
22. $8\frac{1}{3}$.

Page 148.

8. $28\frac{4}{7}$.
9. 63.
10. 81.
11. 70.
12. $113\frac{3}{5}$.
13. $68\frac{4}{7}$.
14. $87\frac{3}{7}$.
15. 60.
16. 63.
17. 12.
18. 12.

19. $2\frac{4}{5}$.
20. $3\frac{7}{11}$.
21. $6\frac{2}{3}$.
22. 9.

Page 149.

9. $\frac{2}{3}$.
10. $1\frac{1}{7}$.
11. $\frac{3}{4}$.
12. $1\frac{1}{3}$.
13. $1\frac{1}{5}$.
14. $\frac{2}{5}$.
15. $1\frac{3}{7}$.
16. $\frac{5}{6}$.
17. $2\frac{2}{3}$.
18. $3\frac{3}{4}$.
19. $5\frac{5}{9}$.
20. $\frac{6}{25}$.
21. $\frac{5}{6}$.
22. 2.
23. $\frac{24}{35}$.
24. $1\frac{1}{24}$.
25. $\frac{4}{5}$.
26. $1\frac{1}{4}$.
27. $1\frac{9}{40}$.
28. $\frac{27}{35}$.
29. $\frac{5}{9}$.
30. $1\frac{4}{5}$.
31. $9\frac{3}{5}$.
32. $10\frac{1}{2}$.

Page 152.

31. $\frac{75}{4}$.
32. $5\frac{13}{20}$.
33. $\frac{72}{120}$, $\frac{100}{120}$, $\frac{105}{120}$.
34. $1\frac{2}{3}$.
35. $152\frac{1}{15}$.
36. $11\frac{11}{15}$.
37. $\frac{9}{14}$.
38. 5536.
39. $\frac{7}{10}$.
40. $46\frac{6}{11}$.
41. $1\frac{5}{16}$.
42. 1650 feet.
43. \$3750.
44. 160 rods.
45. 80 rods.
46. 36 barrels.
47. \$21000.
48. \$3500.
49. 2016 bushels.

Page 153.

50. $\$2533\frac{1}{3}$; $\$1266\frac{2}{3}$.
51. $74\frac{17}{24}$ acres.
52. $2\frac{5}{12}$ yards.
53. $\$41\frac{7}{8}$.
54. $\frac{9}{82}$; \$32000.
55. $\frac{18}{60}$; 60 miles.
56. $\frac{5}{12}$; 288 acres.
57. $\frac{1}{6}$; \$14400.
58. \$64.
59. $131\frac{1}{2}$ pounds.
60. $37\frac{81}{82}$ miles.

Page 154.

61. \$17600.
62. $99\frac{1}{2}$ pounds; $3316\frac{2}{3}$ cents.
63. 13 bushels.

DECIMAL FRACTIONS.

Page 157.

41. .000422
42. .0712
43. .000015
44. .00414
45. .0217

Page 158.

49. 45.52
50. 40.045
51. 200.000079
52. .000279
53. 5000.000066
54. .005066
55. 14.05014
56. .001087
57. .0842
58. 75.00403

Page 161.

6. .4500
7. 6.500
8. 23.00
9. 62.5000
11. .5
12. 2.40

Page 162.

14. $\frac{3}{4}$.
15. $\frac{5}{8}$.
16. $\frac{1}{16}$.
17. $\frac{1}{8}$.
18. $\frac{2}{25}$.
19. $\frac{7}{200}$.
20. $\frac{3}{40}$.
21. $\frac{8}{125}$.
22. $\frac{64}{125}$.
23. $\frac{11}{40}$.
24. $\frac{6}{625}$.
25. $3\frac{1}{4}$.
26. $21\frac{3}{40}$.
27. $4\frac{3}{8}$.
28. $18\frac{3}{4}$.
29. $\frac{1}{800}$.
30. $\frac{2}{625}$.
31. $12\frac{3}{8}$.
32. $25\frac{4}{125}$.
33. $2\frac{1}{16}$.
37. .625
38. .15
39. .25
40. .375
41. .0625
42. .08
43. .075
44. .0625
45. .256
46. .0075
47. .06
48. .015
49. .064
50. 56.25
51. 13.4
52. 30.75
53. 7.08
54. .625
55. .1208
56. 4.0625
57. 15.1875

Page 163.

5. 28.2104
6. 182.097
7. $926.498
8. 178.48 miles.
9. .5378
10. .208647

Page 164.

3. 16.544
4. .032625
5. .011992
6. .7992
7. 31.48
8. .174
9. .0092

Page 165.

8. .00045
9. .02944
10. .09588
11. 2.5331
12. .1728
13. .4355
14. .18468
15. 161.5
16. 1505.52
17. 1.56
18. 1.3332
19. .000056
20. .03625
21. 123.178
22. .021
23. 3.1828
24. .00156

Page 166.

27. 40.85
28. 30048.

Page 167.

9. 108.
10. 1.4
11. .056

Page 168.

12. .0263 +
13. 5.6
14. .69
15. $.026\frac{2}{3}$
17. $533.33\frac{1}{3}$
18. 119.0476 +
19. 75.
20. 7000.
21. 1150000.

22. 3130.
25. .04367
26. .002346

Page 169.

1. .02$\frac{1}{3}$
2. .016$\frac{1}{3}$
3. $\frac{13}{400}$.
4. .064
5. 16.075
6. 99.7125
7. 60.526

Page 170.

8. 255.1632
9. 70470.
10. .00102
11. .0341$\frac{1}{3}$
12. .1743
13. .0072
14. 176000.
15. 40.3 miles.
16. 8.2 hours.
17. 40.4 bushels.
18. .05

UNITED STATES MONEY.

Page 173.

2. $17.825
3. 143.78\frac{1}{2}$
4. $403.575
6. $.49$\frac{3}{4}$, or $.4975
7. $98.744
8. $9.90
9. $327.75
10. 1.37\frac{1}{2}$

Page 174.

11. $2.50
12. 4.87\frac{1}{2}$
13. $493.25
14. $2.75
15. $881.73
2. $120.
3. $56.
4. $144.90
5. 8.43\frac{3}{4}$
6. 109.37\frac{1}{2}$

Page 175.

7. $1200.
8. $106.65
10. $.12
11. $37.50
12. $7.50
13. 11.37\frac{1}{2}$
15. 60 bushels.
16. 210 lemons.
17. 32 days.
18. 148$\frac{3}{8}$ bushels.
19. 75.
20. 36.
21. 220.
22. 875.

Page 176.

1. $139.59
2. $9.995; $495.
3. $506.
4. 100.
5. 250.
6. $131.60
7. $600.
8. $20.
9. $277.20
10. 200 yards.
11. 400 bushels.
12. $1600.

Page 177.

13. $1.75
14. 56 yards.
15. $6.75
16. $288.
17. $1950.
18. $191.25
19. $405.
20. 98 yards.
21. 40 sheep.
22. $935.
23. $284.206
24. $71.

Page 178.

25. $116.95
26. $215.
27. $2.50
28. $39.375
29. $39.15
30. $15.625
1. $13.88

Page 179.

2. $188.645
3. $127.395

Page 180.

4. $132.95
5. $329.
6. $179.25

Page 181.

7. $19.702 +
8. $97.628 +
9. $31.291 +
10. $26.125

DENOMINATE NUMBERS.

Page 187.

18. 768 pt.
20. 987 pt.
21. 417 pt.
22. 63 pt.
23. 366 pt.
24. 971 pt.
25. 280 qt.
26. 31 qt.
27. 37 qt.
28. 705 pt.

Page 188.

30. 5 bu. 1 pk.
31. 5 bu. 1 pk. 3 qt.
32. 5 pk. 5 qt.
33. 2 pk. 6 qt. 1 pt.
34. 3 bu. 3 pk. 5 qt.
35. 544 qt.; *5 bu. 1 pk. 4 qt.*
36. 58 pt.; 12 pk. 4 qt. 1 pt.
37. 673 qt.; 14 bu. 3 qt. 1 pt.
38. $4.88
39. $4.25
40. 4 bu. 3 pk. 3 qt.
41. $1.

Page 190.

15. 672 gi.
16. 431 gi.
17. 793 gi.
18. 185 pt.
19. 251 gi.
20. 77 pt.
21. 4 gal. 3 qt.; 37 qt. 1 pt.
22. 7 gal. 2 qt. 1 pt. 1 gi.
23. 16 qt. 2 gi.
24. 1456 gi.
25. 449 pt.; 17 gal. 1 pt.
26. 38 gal. 1 pt.; 350 pt.
27. 52 gal. 2 qt.; 496 gi.
28. $1.70
29. $28.80
30. 16 vials.

Page 192.

16. 27828 in.
17. 1819 ft.
18. 6490 yd.
19. 320784 in.
20. 5590 rd.
21. 258820 in.
22. 11 mi.
23. 7 mi. 9 rd. $2\frac{1}{2}$ yd.; or, 7 mi. 9 rd. 2 yd. 1 ft. 6 in.
24. 13200 ft.
25. 3247.2 in.
26. 3 mi.

Page 194.

8. 4000 sq. rd.
9. 32670 sq. ft.
10. 196322½ sq. yd.
11. 6368 sq. in.
12. 462916 sq. ft.
13. 33 A.
14. 4 sq. yd.
15. 64 sq. yd. 8 sq. ft.
16. 12 sq. ft.
17. 1 A. 100 sq. rd. 10 sq. yd. 7 sq. ft.
18. 3360 A.

Page 195.

1. 17280 cu. in.
2. 42336 cu. in.
3. 55296 cu. in.
4. 78624 cu. in.
5. 2295 cu. ft.
6. 2601 cu. ft.
7. 1632 cu. ft.
8. 9 cu. ft.
9. 3240 cu. ft.
10. 13 cu. yd.
11. 15 cu. yd.
12. 728793 cu. in.
13. 12 cd.
14. 31 cu. yd. 15 cu. ft. 1206 cu. in.

Page 197.

11. 54000 sec.
12. 8 h.
13. 481200 sec.
14. 2680245 sec.
15. 21 d. 6 h.
16. 6 yr.
17. 527040 min.

Page 198.

18. 31556926 sec.
19. 31536000 sec.
20. 23421 d.; 562104 h.
21. 2208 h.; 2208 h.; 2184 h.; 2160 h., or (l. yr.) 2184 h.
22. 41760 min.; 40320 min.
23. 3780000 times.
24. 13 d.; 1 w. 6 d.

Page 199.

9. 160000 oz.
10. 7456 lb.
11. 9245 oz.
12. 17 T. 9 cwt. 20 lb.
13. 200 cwt. 85 lb.
14. 352608 oz.
15. $11.76; $8.82

Page 200.

16. $90; $72.
17. $507.875

Page 201.

6. 6000 sheets.
7. 3252 sheets.
8. 300 quires; 15 reams.
9. 5184 crayons.

Page 202.

10. 480 shirts.
11. $176.
12. $22.50

Page 204.

2. 89 bu. 3 pk. 5 qt.
3. 121 gal. 3 qt. 1 pt.
4. 98 mi. 305 rd. 2 ft. 11 in.
5. 29 w. 6 d. 10 h. 41 min. 29 sec.

Page 205.

6. 3 sq. mi. 229 A. 4 sq. rd.
7. 11 cwt. 61 lb. 10 oz.
8. 18 rd. 3 yd. 1 ft. 7 in.

Page 206.

2. 19 cwt. 25 lb. 12 oz.
3. 17 gal. 3 qt. 1 pt. 3 gi.
4. 4 w. 1 d. 20 h. 12 min.
5. 2 bu. 5 qt. 1 pt.
7. 9 yr. 7 mo. 22 d.
8. 4 yr. 1 mo. 10 d.
9. 56 yr. 2 mo. 3 d.
10. 49 yr. 10 mo.
11. 7 yr. 9 mo. 1 d.
12. 283 yr. 8 mo. 20 d.
13. 2 yr. 9 mo. 12 d.
14. March 15, 1767

Page 207.

2. 251 bu. 5 qt.
3. 34 w. 3 d. 19 h.
4. 902 mi. 200 rd. 5 yd. 2 ft. 6 in.
5. 38 yr. 11 mo. 20 d.
6. 98 rd. 3 yd. 2 ft.
7. 24 mi. 246 rd.
8. 87 bu. 3 pk

Page 208.

2. 3 cwt. 28 lb. 15⅓ oz.
3. 4 rd. 3 yd. 1 ft.
4. 19 cwt. 98 lb.
5. 45 lengths.
6. 44 castings.
7. 4 baskets.
8. 52 bottles.

Page 211.

43. 440 qt.
44. 3024 pt.
45. 29216 yd.
46. 29040 sq. yd.
47. 45 cu. yd.
48. 40320 min.; 41760 min.
49. $54.
50. $11.40
51. $85.25
52. $11200.
53. $507.875

Page 212.

54. 61 lb.
55. 28 h. 53 min. 20 sec.
56. 1200 times.
57. 126720 times.
58. 1584 steps.
59. 3520 times.
60. 12 mi. 123 rd. 3½ yd.
61. 240 rd.
62. 2592000 times; 2678400 times.
63. 7305 h.
64. 320 lb.
65. 120 gross.
66. 41⅔ reams.
67. $22.50
68. $36.60

Page 213.

69. $48.30
70. $56.70
71. $162.50; $37.50
72. $14.093 +
73. 5091⅞ steps.
74. 21 jugs.
75. $10.
76. 5 lb. 11⅗ oz.
77. 36 gal. 1 qt. 1 pt. 1⅛ gi.
78. 2450 gal.
79. 19 lb. 4 oz.
80. 61 yr. 7 mo. 2d.
81. 16 yr. 2 mo. 18 d.

Page 214.

82. 7 T. 2 cwt. 50 lb 5 T. 12 cwt. 50 l
83. 2 cwt. 54 lb. 4 oz.
84. 2 reams 5 quir.
85. 43 cd. 80 cu. ft.
86. 2 T. 17 cwt. 10 lb.; $24.267 +

MENSURATION.

Page 217.

12. $166\frac{2}{3}$ sq. yd., or 166 sq. yd. 6 sq. ft.
13. 14 A.
14. $22\frac{1}{2}$ A.
15. 50 A.
16. 320 A.; 160 A.
17. 240 A.

Page 218.

1. 63 yd.
2. 32 yd.; 32 yd.
3. $28\frac{1}{3}$ yd.
4. $47\frac{5}{6}$ yd.
5. 80 yd.; 140 yd.

Page 219.

6. $74\frac{4}{9}$ sq. yd.
7. $91\frac{1}{9}$ sq. yd.
8. \$18.537+
9. \$14.847 +
10. \$25.394+

Page 220.

12. 512 ft.
13. $213\frac{1}{3}$ ft.
14. 7 ft.
15. 45 ft.
16. 301 ft.
17. 480 ft.

Page 222.

12. 1872 cu. ft.
13. 1152 cu. ft.
14. 220 cu. yd.
15. $83\frac{1}{3}$ cu. ft.
16. $19\frac{4}{9}$ cu. ft.
17. 200 cu. ft.
18. $\$63.33\frac{1}{3}$.

Page 223.

1. $13\frac{1}{2}$ cd.
2. $3\frac{1}{8}$ cd.
3. 9 cd.
4. 15 cd.
5. $5\frac{1}{4}$ cd.; $5\frac{5}{8}$ cd.

Page 224.

6. $4\frac{1}{2}$ cd.
7. \$23.625
8. $213\frac{1}{3}$ perches.
9. 57 cu. yd.
11. 36 bu. common measure, or 36.16 + bu. accurate.

Page 225.

8. $\$5333.33\frac{1}{3}$.
9. 23040 A.
10. \$600.
11. 24200 hills.
12. 12800 persons.
13. 480 trees.

Page 226.

14. \$125.
15. \$10642.50
16. 120 sq. yd.
17. $72\frac{1}{2}$ sq. yd.
18. $30\frac{5}{9}$ yd.
19. 14400 shingles, or $14\frac{2}{5}$ M.
20. $12\frac{1}{8}$ rd.
21. 336 cu. ft.
22. \$40.50
23. \$406.25
24. 54 bu. common measure, or 54.24+ bu. accurate.
25. 411.428+ gal.

PERCENTAGE.

Page 230.

19. $260.
20. 14.7
21. 108.
22. 96.6
23. 77.
24. 105 ft.
25. 184.8 lb.
26. 10.7 oz.
27. 120 men.
28. $2.25
29. $3.15
30. 2.43⅓ days.
31. $28.95
32. $.0375
33. .516
34. 1.215 ft.
35. 6.118 mi.
36. 2.64 A.
37. 7.84 lb.
38. 48 d.
39. 3.5 ft.
40. 90 yd.
41. $2.31
42. $.30
43. .9 pt.
44. 48 rd.
45. $19.
46. $7.15
47. 1859 men.
48. $1587.50
49. *95 A.; 190 A.*
50. 147 sheep.
51. 88 girls.
52. 416 bu.

Page 231.

9. 10%.
10. 25%.
11. 18%.
12. 7½%.

Page 232.

13. 2½%.
14. 11⅓%.
15. 6%.
16. 5%.
17. 40%.
18. 15%.
19. 5%.
20. 14%.
21. 12½%.
22. 4½%.
23. 15%.
24. 110%.
25. 10%.
26. 105%.
27. 85%.
28. 40%.
29. 80%.
30. 25%.
31. 8%.
32. 8%.
33. 40%.
34. 16⅔%.
35. 88%.
36. 8%.

Page 233.

10. 200.
11. $462.
12. $56.
13. $65.28
14. $216.
15. 31.
16. 6.93
17. 11.2 bu.
18. 95 T.
19. $292.
20. $97.
21. 70 pupils.

Page 235.

25. 264 pupils.

Page 236.

26. 1029 bu. wheat.
588 bu. oats.
833 bu. corn.
27. $13430.
28. 2800 lb.
29. $54000.
30. $70.
31. $1500.
32. 70%.

33. 15%.
34. 80%.
35. 30%.
36. $12250.
37. $21692.
38. $175.50
39. $7\frac{1}{2}$%.
40. $1.12\frac{1}{2}$; $1.31\frac{1}{4}$; $1.37\frac{1}{2}$.

Page 237.

41. $175.
42. $32.45
43. $635.
44. $93.60
45. $41.
47. $18.75
49. $62.
50. $12225.
51. $513.
52. $11750.

INTEREST.

Page 239.

13. $15.
14. $15.30
15. $20.80
16. $11.20
17. $10.254
18. $150.25
19. $11.55
20. $5.76
21. $22.14
22. $40.
23. $9.24
24. 85.33\frac{1}{3}$
25. $38.44

Page 240.

26. $.56$\frac{1}{4}$
27. $.98
28. $16.80
29. $1.60

Page 241.

12. $.105
13. $.129$\frac{1}{6}$
14. $.198$\frac{1}{3}$
15. $.069
16. $.130$\frac{1}{3}$
17. $.065$\frac{1}{6}$
18. $.036$\frac{1}{6}$
19. $.300$\frac{5}{6}$

Page 242.

23. (2) $19.1464; (3) $18.0544
24. $4.815; $1.7655
25. $5.571+; 20.66\frac{1}{4}$
26. $9.1476; $22.946
27. $66.11; $28.5475
28. $16.933+; $42.076+
29. $34.; $11.25
30. $15.6492; $13.797
31. $17.25; $58.65
32. $5.742+; $11.8179+
33. $4.161; $9.417
34. $.6029+; $2.962+
35. $.273+; $1.545+
36. $13.033+; $7.50

Page 243.

38. $117.60
39. $6.8306
40. $20.758
41. $.864+
42. $21.129+
43. $1.189+
44. $118.41+
45. $12.507+
46. $301.44
47. $203.765
48. $148.442+
49. $164.7198
50. $215.908+

51. $524.40
52. $302.9625

Page 246.

2. $22.263 +
3. $714.72
4. $61.568
5. $122.578 +
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11. $221.85
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1. $458.16
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3. $3.024
4. $.084; $.108; $.12

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To TEACHERS.—There is great advantage in having many problems in an elementary arithmetic without answers accessible to pupils; and it is not necessary for the teacher to be in possession of the answers, or, in their absence, to solve the problems. The following plan of testing the accuracy of the answers obtained by pupils, has been successfully used by many teachers:

When problems, the answers to which are not given, are recited in class, the teacher calls on a pupil to give his answer, and all the members of the class, who have the same answer, indicate this by raising the hand, or otherwise. If *all agree*, the answer given may be accepted as correct, provided the problem has no ambiguity in statement or other special difficulty. If a majority of the pupils agree, and the other pupils do not agree *with each other*, the answer obtained by the majority may usually be accepted as correct. When the pupils do not agree as to the answer, an analysis of the problem or a statement of the steps of the solution will disclose the error made, and the true answer *will then* be obtained by the pupils. A little attention each day *to the written solutions* of one or more members of the class will *secure honesty in* reporting answers.

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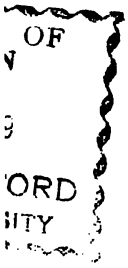

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